Sample Calculation of Induced Module

Step 1: Start with the 2-dimensional module W for S_3 with representing matrices:

$$\psi(e) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \psi((1,2)) = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}, \psi((2,3)) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\psi((1,3)) = \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix}, \psi((1,2,3)) = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}, \psi((1,3,2)) = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$

Step 2: Choose a set of coset representatives for S_3 in S_4:

$$\{e = g_1, (1,4) = g_2, (2,4) = g_3, (3,4) = g_4\}$$

Step 3: Let $\sigma = (1,2)(3,4)$. We will compute $(\operatorname{Ind}\psi)(\sigma)$. We compute the values $g_j^{-1}\sigma g_i$ giving:

$$\begin{pmatrix} (1,2)(3,4) & (1,3,4,2) & (1,2,3,4) & {\color{red}(1,2)} \\ (1,2,4,3) & (1,3)(2,4) & {\color{red}(1,2,3)} & (1,2,4) \\ (1,4,3,2) & {\color{red}(1,3,2)} & (1,4)(2,3) & (1,4,2) \\ {\color{red}(1,2)} & (1,2,4) & (1,4,2) & (1,2)(3,4) \end{pmatrix}$$

Only those in red are in S_3.

Step 4: The $\dot{\psi}(g_j^{-1}\sigma g_i)$ matrix is:

$$(\operatorname{Ind}\psi)(1,2)(3,4) = \begin{pmatrix} & & & & & & -1 & -1 \\ & & & & & & 0 & 1 \\ & & & & -1 & -1 & & \\ & & & & 1 & 0 & & \\ & & 0 & 1 & & & & \\ & & -1 & -1 & & & & \\ -1 & -1 & & & & & & \\ 0 & 1 & & & & & & \end{pmatrix}$$

Notice the induced character vanishes on $\sigma = (1,2)(3,4)$, as no conjugate of σ lies in S_3.