Math 561 Fall 2013 Homework Number 7

Due Wednesday November 20, 2013

1. Problem 5.16.2

2. Problem 5.16.3

3. Consider the Specht module $S^{(4,1,1)}$. Calculate its dimension. Determine the Gram matrix for the bilinear form on $S^{(4,1,1)}$ in the basis of standard polytabloids. Calculate its rank in characteristic 3 (you may use a computer package or row reduction, I don't need to see your work). Conclude by determining the dimension of the simple module $D^{(4,1,1)}$ in characteristic 3.

4. Find the standard basis for the Specht module $S^{(3,1)}$ (it is 3 dimensional) and write down representing matrices for $(1,2), (1,2,3), (1,2)(3,4)$ and $(1,2,3,4)$. This corresponds to a homomorphism:

$$\rho : S_4 \to GL(V)$$

where $V \cong \mathbb{C}^3$. Recall from class that $L_{(2,1)}(V)$ is a $GL(V)$ module, and hence an S_4 module. Decompose the character of this module into irreducibles for S_4. Repeat for $L_{(2,2)}(V)$.