Math 561 Fall 2013 Homework Number 6

Due Monday November 4, 2013

1. Use the hook length formula to compute the dimensions of the irreducible modules for S_6. Verify the sum of the squares is $6!$.

2. Let $\sigma \in S_n$ be an n-cycle and $\lambda \vdash n$. Use the Frobenius formula to prove $\chi_\lambda(\sigma) = 0$ unless λ is a hook partitions (i.e. of the form $(n-d, 1^d)$). In this case show $\chi_\lambda(\sigma) = (-1)^d$.

3. Problem 5.24.1

4. Recall $V_\lambda = \mathbb{C}S_n c_\lambda = \mathbb{C}S_n a_\lambda b_\lambda$ is irreducible.

a) Prove that $V_\lambda \cong \mathbb{C}S_n b_\lambda a_\lambda$. (Hint: Use Schur's Lemma and the fact that right multiplications are left module homomorphisms.)

b) Conlude that V_λ is the image of the map from $\mathbb{C}S_n a_\lambda$ to $\mathbb{C}S_n b_\lambda$ given by right multiplication by b_λ, and similarly the other way.

c) Prove that $V_{\lambda'} \cong V_\lambda \otimes \mathrm{sgn}$.