Math 561 Fall 2013 Homework Number 3

Due Monday September 30, 2013

1. Let $M_n(k)$ be the algebra of $n \times n$ matrices with entries in k and let P be an invertible element of $M_n(k)$.

a) Prove that $\psi(B) = PBP^{-1}$ is an algebra automorphism of $M_n(k)$. Such automorphisms are called *inner.*

b) Recall from class that $M_n(k)$ has a unique simple module, namely column vectors k^n. Use this fact to prove that any automorphism of $M_n(k)$ is inner.

2. Let U_1, U_2 be submodules of an A-module U.

a) Let $U_1 + U_2 = \{u_1 + u_2 \mid u_1 \in U_1, u_2 \in U_2\}$. Prove that $U_1 + U_2$ is a submodule of U. Prove that $U_1 \cap U_2$ is a submodule of U.

b) Under what circumstance is $U_1 + U_2 \cong U_1 \oplus U_2$?

c) Suppose $S_1, S_2, \ldots, S_t$ are simple submodules of U. Prove that $S_1 + S_2 + \cdots + S_t$ is semisimple. Hint: There is an obvious surjection from $S_1 \oplus S_2 \oplus \cdots \oplus S_t$. Now consider Prop 3.1.4.

3. Problem 3.6.1

4. Problem 3.9.1