The D_4 root system

The D_l root system can be realized as the set of vectors in $\mathbb{R}^l$ with integer coordinates and length $\sqrt{2}$. It clearly has $4 \cdot \binom{n}{2} = 2n(n-1)$ roots. A choice of simple roots is $\alpha_i = e_i - e_{i+1}$ for $1 \leq i < l$ together with $e_{l-1} + e_l$. For example in type D_4 we have:

$$\begin{aligned}
\alpha_1 &= (1,-1,0,0) \\
\alpha_2 &= (0,1,-1,0) \\
\alpha_3 &= (0,0,1,-1) \\
\alpha_4 &= (0,0,1,1)
\end{aligned}$$

The standard labelling of the Dynkin Diagram for D_l is:

$$\begin{array}{ccccccccc}
 & & & & & & \circ & & \\
 & & & & & & | \; \alpha_l & & \\
\circ & — & \circ & — & \cdots & — & \circ & — & \circ \\
\alpha_1 & & \alpha_2 & & & & \alpha_{l-2} & & \alpha_{l-1}
\end{array}$$

We have the following 12 positive roots:

$$\begin{aligned}
\alpha_1 &= (1,-1,0,0) \\
\alpha_2 &= (0,1,-1,0) \\
\alpha_3 &= (0,0,1,-1) \\
\alpha_4 &= (0,0,1,1) \\
\alpha_1 + \alpha_2 &= (1,0,-1,0) \\
\alpha_2 + \alpha_3 &= (0,1,0,-1) \\
\alpha_2 + \alpha_4 &= (0,1,0,1) \\
\alpha_1 + \alpha_2 + \alpha_3 &= (1,0,0,-1) \\
\alpha_2 + \alpha_2 + \alpha_4 &= (1,0,0,1) \\
\alpha_2 + \alpha_3 + \alpha_4 &= (0,1,1,0) \\
\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 &= (1,0,1,0) \\
\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 &= (1,1,0,0)
\end{aligned}$$

Note the correspondence between the positive roots and the dimension vectors for the indecomposable modules for the D_4 quiver.