Math 464/564 Fall 2017 Homework Number 7- Due Tuesday 10/24/17

1. Sagan 2.12 #10.

2. Sagan 2.12 # 11.

3. Sagan 2.12 # 12.

4. Sagan 2.12 #13 (Hint: Use #12).

5. Let V be a vector space over a field k. Recall the dual space V^* is the set of linear functionals on V, i.e. linear maps from V to the field k. Suppose further that V is a kG module. Define an action on V^* by:

$$(g \cdot \psi)(v) := \psi(g^{-1}v).$$

a) Prove this makes V^* into a left G module.

b) Suppose $k = \mathbb{C}$. Describe the character χ_{V^*} in terms of the character χ_V.

c) Now suppose $V = S^{(2,1)}$ and k is a field of characteristic 3, (for example $\mathbb{Z}/3\mathbb{Z}$). Prove that $S^{(2,1)}$ is not isomorphic to its dual.