Math 464/564 Fall 2017 Homework Number 4- Due 9/26/17

1. Sagan 1.13 # 5.

2. Sagan 1.13 #15

3. Sagan 1.13 #17.

4. Suppose χ is a nonzero, nontrivial character of G such that $\chi(g)$ is a non-negative real number for all $g \in G$. Prove that χ is not irreducible.

5. Recall the dihedral group D_8 can be presented as $D_8 = \{r, s \mid r^4 = s^2 = e, sr = r^3 s\}$.

a) Show $Z(D_8) = \{e, r^2\}$ and $D_8/Z(D_8)$ is isomorphic to the Klein four group V.

b) Determine the character table for V.

c) Use the character table for V to construct four distinct linear characters of D_8.

d) Use the orthogonality relations to complete the character table of D_8.

6.

a) Let X be an irreducible representation of G with character χ. Show $\sum_{g \in G} \chi(g) = 0$ unless X is the trivial representation.

b) Let $H \leq G$ and $g \in G$ be such that all the elements of the coset gH are in the same conjugacy class of G. Let χ be a character of G such that $\langle \chi_H, 1_H \rangle = 0$. Show that $\chi(g) = 0$. Hint: Let $A = \sum_{h \in H} X(hg)$. Compute the trace of A.

Remark: If $X : G \to GL_d$ is a representation of G and $H \leq G$ then $X : H \to GL_d$ is a representation of H, known as the *restriction* of X to H. So χ_H denotes the character of this restricted representation.