Math 464/564 Fall 2017 Homework Number 1- Due 9/5/17

1. Let $\sigma = (1,6,2,4)(3,7,8)(5)$ and $\tau = (1,4,6,8)(3,5)(2,7)$ be permutations in S_8.

a) Calculate $\sigma\tau$, $\tau\sigma$, and $\tau^{-1}\sigma\tau$.

b) Calculate the order of σ and τ.

c) What is the largest possible order of a permutation in S_{10}? Explain.

2.

a) Consider σ given in two-line notation by:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 9 & 5 & 6 & 1 & 10 & 4 & 7 & 2 & 8 \end{pmatrix}$$

Write σ in disjoint cycle notation.

b) Let $\tau = (12)(34)(5,6)(7,8,9)(10,11,12)(13,14,15,16)$. How many ways are there to write the same permutation τ in the form $(??)(??)(??)(???)(???)(????)$. For example

$$\tau = (65)(34)(21)(8,9,7)(12,10,11)(15,16,13,14).$$

3. Let V be the vector space of polynomials with real coefficients and degree at most 3. So $\{1, x, x^2, x^3\}$ is a basis of V. Let $\beta = \{v_1 = 1+x, v_2 = 1-x, v_3 = x+x^2+x^3, v_4 = x^2+2x^3.\}$

a) Prove that β is also a basis of V.

b) Consider the map T that takes a polynomial $p(x)$ to its derivative $p'(x)$. Prove that T is a linear map from V to V.

c) Find the matrix of T with respect to the basis β.

4. Let G be a group and $g \in G$. The *centralizer* of g, denoted $C_G(g)$ is the set of elements which commute with g. That is:

$$C_G(g) = \{x \in G \mid xg = gx\}.$$

a) Prove that $C_G(g)$ is a subgroup of G.

b) Let $G = S_4$ and $\sigma = (12)(34)$. Determine $C_G(\sigma)$.

c) Let $G = GL_2(\mathbb{R})$ and $g = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Determine $C_G(g)$.

5. Let $G = S_4$ and let $H = \langle(123)\rangle$ be the subgroup generated by (123). Compute the left cosets of H in G.

6. Recall that for groups G and H a map $\psi : G \to H$ is a *homomorphism* if $\psi(g_1g_2) = \psi(g_1)\psi(g_2)$ for all $g_1, g_2 \in G$. Recall that the *kernel* $\ker\psi = \{g \in G \mid \psi(g) = e\}$. Prove that the kernel is a subgroup of G.