Math 461/561 Homework #1: Due Tuesday September 7

0. Read Chapter 1.

1. Show that $\beta = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis of $\mathbb{R}^3$.

2. Check that $T(x, y, z) = (2x + y, x + z)$ is a linear map from $\mathbb{R}^3 \to \mathbb{R}^2$. Then write the matrix of T in terms of the standard bases.

3. Let T be a linear map on $\mathbb{R}^2$ with $T(2, 1) = (5, 2)$ and $T(1, 2) = (7, 10)$. Determine the matrix of T with respect to the basis $\beta = \{(3, 3), (1, -1)\}$.

4. Let
$$A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & 5 \\ -1 & -2 & 1 \end{pmatrix}.$$
Determine the characteristic polynomial of A.

5. Let $T : U \to V$ and $S : V \to W$ be linear transformations. Prove that the composition $S \circ T : U \to W$ is a linear transformation.