Math 353 Homework #8- Due Wednesday 11/2/16

1. Let $\sigma = (1, 2, 3)(4, 5, 6)(7, 8)(9, 10, 11, 12)$ and $\tau = (1, 2, 3, 4)(5, 6, 7)(8, 9, 10, 11, 12)$.
   - Find $\sigma\tau$, $\tau\sigma$, τ^{-1}, and $\tau\sigma\tau^{-1}$.
   - The *order* of an element g is the smallest nonnegative integer n so that $g^n = e$. What is the order of an n-cycle $(1, 2, 3, 4, \ldots, n)$.
   - Calculate the order of σ and τ.
   - Write σ in 1-line notation. Is it 231-avoiding?

2. Prove that every group with 4 elements is isomorphic to either the group Z_4 or the Klein group V.

3. 11.3.2B

4. 11.4.1B

5. 11.5.3.B

6. Calculate the orders of all the elements of the group $Z_{12} = \{0, 1, 2, \ldots, 11\}$ under addition modulo 12.

7. Find a 2×2 invertible matrix which has order 3. Find one which has infinite order.

8. For a group G, the center $Z(G)$ is defined as the elements that commute with every other element. That is:

$$Z(G) = \{z \in G \mid zg = gz \ \forall g \in G\}.$$

Note that $e \in Z(G)$ so is is nonempty. Prove that $Z(G)$ is always a subgroup, and find the center of the group of symmetries of the square.