Math 353 Homework #6- Due Friday 10/15/16

1. 6.2.3B. Hint: Consider solutions to equations.

2. 6.2.4B

3. Write down all partitions of 20 into parts all congruent to $\pm 1 \bmod 5$. (In other words you can only use $1, 4, 6, 9, 11, 14, 16, 19$). Now write down all partitions whose parts differ by at least two: $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)$ with $\lambda_i - \lambda_{i+1} \geq 2$ for all i (the last part may be 1). You should get the same number of partitions.

Repeat this for partitions with all parts congruent to $\pm 2 \bmod 5$ (so you can use 2,3, 7,8,12,13,17,18) and partitions whose parts differ by at least two and that have no part equal to 1. Again you should get the same number.

That these numbers agree are the famous Rogers-Ramanujan identities. The results are known but if you can come up with a nice combinatorial proof you will be famous!

4. Prove the number of partitions of n into powers of 2 is even.

5. Prove the number of partitions of n into distinct parts is equal to the number of partitions of n into odd parts.

6. 6.4.2B

7. Consider the following three permutations in S_{12} in one-line notation:

$\pi = 9, 3, 1, 4, 7, 5, 2, 11, 10, 12, 6, 8 \quad \sigma = 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 \quad \gamma = 12, 1, 11, 2, 10, 3, 9, 4, 8, 5, 7, 6$

Run the RSK correspondence on each and give the corresponding pair of standard young tableaux.

8. Consider the pair of standard tableaux given below, both of shape $(4, 3, 2, 2)$. Find the permutation $\pi \in S_{11}$ that outputs this pair under the RSK correspondence.

$$P = \begin{array}{cccc} 1 & 3 & 4 & 10 \\ 2 & 6 & 7 & \\ 5 & 8 & & \\ 9 & 11 & & \end{array} \qquad Q = \begin{array}{cccc} 1 & 2 & 6 & 10 \\ 3 & 5 & 8 & \\ 4 & 9 & & \\ 7 & 11 & & \end{array}$$