Name: SOLUTIONS

241S1 Quiz #6 - October 20, 2015, 10 a.m.

1. Let $f(x, y)$ be a function. Define what it means for (a, b) to be a **critical point** for $f(x, y)$.

$\nabla f(a,b) = (0,0)$ or $\nabla f(a,b)$ DNE

2. Let $f(x, y) = 2x^3 + y^4$. Let $D = \{x^2 + y^2 \leq 1\}$. Find the absolute maximum and minimum values of $f(x, y)$ on the set D.

$\nabla f = (6x^2, 4y^3)$ so only c.p. is $(0,0)$

Boundary is $y^2 = 1 - x^2$ $\quad -1 \leq x \leq 1$ so $f(x,y) = 2x^3 + (1-x^2)^2$
$= 2x^3 + x^4 - 2x^2 + 1$

Set $f' = 0$: $\quad f'(x) = 4x^3 + 6x^2 - 4x$
$= 2x(2x^2 + 3x - 2) = 2x(2x-1)(x+2)$
for $-1 \leq x \leq 1$

so check $x = 0$, $x = 1/2$ and endpoints $x = \pm 1$!

Note $x = -2$ not in domain.

Point	$f(x,y)$
$(0,0)$	0
$(0,1)$	1
$(0,-1)$	1
$(1/2, \sqrt{3}/2)$	$13/16$
$(1/2, -\sqrt{3}/2)$	$13/16$
$(1,0)$	2
$(-1,0)$	-2

Global max value 2 at $(1,0)$
Global min value -2 at $(-1,0)$

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241S3 Quiz #6 - October 22, 2015, 10 a.m.

1. Let $f(x, y) = x^2 + y^4 + 2xy$. Find the critical points. Use the second derivative test to classify each as a local max, min or saddle point.

$\nabla f = (2x+2y, 4y^3+2x)$ set $= (0,0)$

$2x+2y=0 \Rightarrow y=-x$

$\Rightarrow -4x^3+2x=0 \Rightarrow 2x(1-2x^2)=0$

$x=0, x=\pm 1/\sqrt{2}$

$f_{xx}=2 \quad f_{xy}=2$

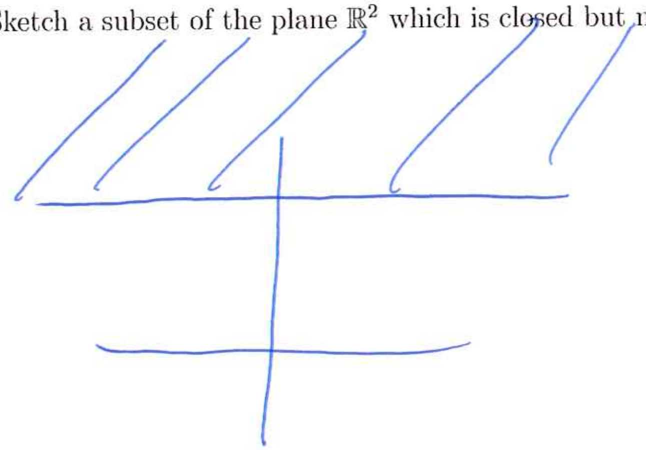
$f_{yy}=12y^2$

$D=24y^2-4$

3 critical points

	D	f_{xx}	
$(0,0)$	-4		saddle
$(1/\sqrt{2}, -1/\sqrt{2})$	8	2	min
$(-1/\sqrt{2}, 1/\sqrt{2})$	8	2	min

2. Sketch a subset of the plane $\mathbb{R}^2$ which is closed but not bounded.



$\{y \geq 3\}$

many possible answers

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241S2 Quiz #6 - October 22, 2015, 11 a.m.

1. Let $f(x,y) = xy$. Use Lagrange multipliers to find the extreme values of $f(x,y)$ subject to the constraint $4x^2 + y^2 = 8$.

$\nabla f = (y, x)$

$\nabla g = (8x, 2y)$

$y = 8\lambda x$

$x = 2\lambda y$

$4x^2 + y^2 = 8$

$\Rightarrow y = 16\lambda^2 y$

$y = 0$ means $x = 0$ not on g

else $\lambda = \pm 1/4$

$\lambda = 1/4 \quad y = 2x$

$8x^2 = 8 \quad x = \pm 1$

$y = \pm 2$

$\lambda = -1/4 \quad y = -2x \quad x = \pm 1 \; y = \mp 2$

4 points

$(1,2)$

$(1,-2)$

$(-1,2)$

$(-1,-2)$

$\boxed{\begin{array}{l}\text{max value } 2 \text{ at } (1,2), (-1,-2) \\ \text{min value } -2 \text{ at } (-1,2), (1,-2)\end{array}}$

2. Find the equation of the tangent plane to $x^2 + 2y^2 - 3z^2 = 17$ at the point $(4, 2, 1)$.

$\nabla F = (2x, 4y, -6z)$

$\nabla f(4,2,1) = (8, 8, -6)$

$8x + 8y - 6z = 42$