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SOLUTIONS

241S1 Quiz #5 - October 13, 2015, 10 a.m.

1. Let $z = x^3 + x^2y$, $x = s - 2t - u$, $y = stu^2$. Find the partial derivative $\frac{\partial z}{\partial u}$ when $s = 2$, $t = 2$, $u = 1$.

at point, $x = -3$ $y = 4$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u} = (3x^2 + 2xy)(-1) + x^2(2st)$$

$$(27 + (-24))(-1) + 9(8) = -3 + 72 = \textcircled{69}$$

2. Suppose $e^z = xyz$. Find $\partial z / \partial x$.

$$F(x, y, z) = e^z - xyz = 0 \quad \text{so}$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \frac{-yz}{e^z - xy}$$

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2. Suppose $e^z = xyz$. Find $\partial z / \partial x$.

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241S3 Quiz #5 - October ~~10~~¹⁵, 2015, 10 a.m.

1. Let $f(x, y, z) = x^2y + \sin(xz) + y^3$. Find ∇f .

$$\nabla f = (2xy + z \cos(xz), x^2 + 3y^2, x \cos(xz))$$

2. For $f(x, y, z)$ as above, find the maximum rate of change of f at the point $(2, 3, 0)$ and the direction in which it occurs.

$$\nabla f(2, 3, 0) = (12, 31, 2) \quad \text{so max rate of change}$$

$$\text{is } |\nabla f(2, 3, 0)| = \sqrt{12^2 + 31^2 + 2^2} = \sqrt{1109}$$

$$\text{Direction is } (12, 31, 2)$$

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241S2 Quiz #5 - October 15, 2015, 11 a.m.

1. Let $f(x, y) = 4y\sqrt{x}$. Find the directional derivative of $f(x, y)$ at the point $(1, 4)$ in the direction $(3, 2)$.

$$\nabla f = \left(\frac{2y}{\sqrt{x}}, 4\sqrt{x} \right) \quad \nabla f(1, 4) = (8, 4)$$

$$\vec{u} = \left(\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right)$$

$$(8, 4) \cdot \left(\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right) = \frac{32}{\sqrt{13}}$$

2. Find the equation of the tangent plane to $x = y^2 + z^2 + 1$ at $(3, 1, -1)$.

Level curve $x - y^2 - z^2 = 1$ $F(x, y, z) = x - y^2 - z^2$

$$\nabla F = (1, -2y, -2z) \quad \nabla F(3, 1, -1) = (1, -2, 1)$$

$$x - 2y + z = 0$$

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2. Find the equation of the tangent plane to $x = y^2 + z^2 + 1$ at $(3, 1, -1)$.