Name: SOLUTIONS

241S1 Quiz #8 - November 3, 2015, 10 a.m.

1. Let D be the top half of the disc with center at the origin and radius five. Evaluate

$$\iint_D x^2 y \, dA$$

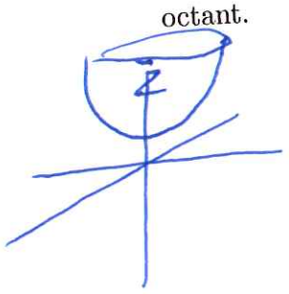
by changing to polar coordinates.

$$\int_0^{\pi} \int_0^5 r^2 \cos^2\theta \cdot r\sin\theta \cdot r \, dr \, d\theta = \int_0^{\pi} \frac{r^5}{5} \cos^2\theta \sin\theta \Big|_{r=0}^{5} d\theta$$

$$= \int_0^{\pi} 625 \cos^2\theta \sin\theta \, d\theta = -\frac{625}{3} \cos^3\theta \Big|_0^{\pi}$$

$$= \frac{625}{3} - -\frac{625}{3} = \boxed{1250/3}$$

2. Find the volume bounded by the parabaloid $z = 1 + 2x^2 + 2y^2$ and the plane $z = 7$ in the first octant.



region in xy plane is $x^2+y^2 = 3$

$$\int_0^{\sqrt{3}} \int_0^{2\pi} \int_{1+2x^2+2y^2}^{7} 1 \, r \, dz \, d\theta \, dr$$

$$= \int_0^{\sqrt{3}} \int_0^{2\pi} (6 - 2r^2) \, r \, dr \, d\theta$$

$$= \int_0^{\sqrt{3}} 2\pi r (6 - 2r^2) \, dr = \int_0^{\sqrt{3}} 12\pi r - 4\pi r^3 \, dr$$

$$= 6\pi r^2 - \pi r^4 \Big|_0^{\sqrt{3}}$$

$$= 18\pi - 9\pi = \boxed{9\pi}$$

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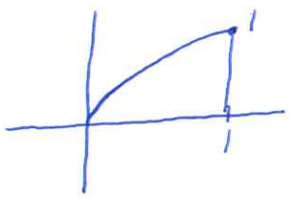
241S3 Quiz #8 - November 5, 2015, 10 a.m.

1. Evaluate

$$\iiint_E 6xy\,dV$$

where E lies under the plane $z = 1 + x + y$ and above the region in the xy plane bounded by the curves $y = \sqrt{x}$, $y = 0$ and $x = 1$.



$$\int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy\,dz\,dy\,dx = \int_0^1 \int_0^{\sqrt{x}} 6xy + 6x^2y + 6xy^2\,dy\,dx$$

$$= \int_0^1 3xy^2 + 3x^2y^2 + 2xy^3 \Big|_{y=0}^{\sqrt{x}}\,dx$$

$$= \int_0^1 3x^2 + 3x^3 + 2x^{5/2}\,dx$$

$$= x^3 + \frac{3}{4}x^4 + \frac{4}{7}x^{7/2}\Big|_0^1$$

$$= 1 + \frac{3}{4} + \frac{4}{7} = \frac{28+21+16}{28} = \frac{65}{28}$$

2. Describe in words the surface given by the equation $(\rho - 2)(\rho - 5) = 0$ in spherical coordinates.

$\rho = 2$ or $\rho = 5$.

Sphere of radius 2 center at $(0,0,0)$

Sphere of radius 5 center at $(0,0,0)$

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241S2 Quiz #8 - November 5, 2015, 11 a.m.

1. Evaluate

$$\iiint \sqrt{x^2+y^2}\,dV$$

where where E is the region that lies inside the cylinder $x^2+y^2=16$ and between the planes $z=-5$ and $z=4$. Use cylindrical coordinates.

$$\int_0^4 \int_0^{2\pi} \int_{-5}^{4} r \cdot r\,dz\,d\theta\,dr = \int_0^4 \int_0^{2\pi} 9r^2\,d\theta\,dr$$

$$= \int_0^4 18\pi r^2\,dr$$

$$= 6\pi r^3 \Big|_0^4$$

$$= 256\pi$$

2. Sketch the region in $\mathbb{R}^3$ given by the equation $\phi = \pi/4$ in spherical coordinates.