Name: SOLUTIONS

241S1 Quiz #7 - October 27, 2015, 10 a.m.

1. Calculate

$$\iint_R xy^2\, dA$$

where $R = \{(x, y) \mid 0 \le x \le 2, 1 \le y \le 2\}$.

$$\int_0^2 \int_1^2 xy^2 \cancel{dA}\, dy\, dx = \int_0^2 \tfrac{1}{3}xy^3\Big|_1^2 dx = \int_0^2 \tfrac{1}{3}x(8-1)$$

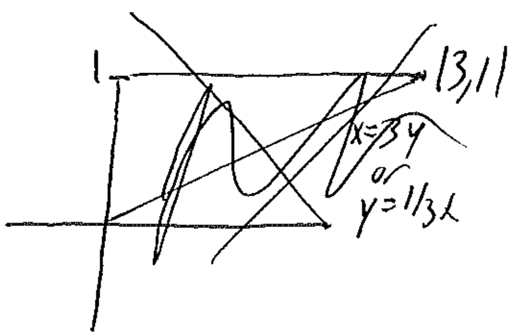
$$= \frac{7}{3}\int_0^2 x\, dx$$

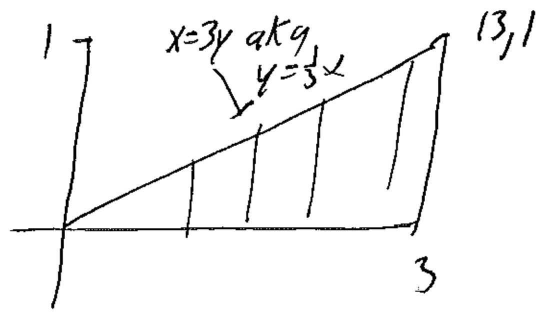
$$= \frac{7}{6}x^2\Big|_0^2 = 28/6$$

$$= 14/3$$

2. Evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$





Reverse: $0 \le x \le 3$

$0 \le y \le x/3$

$$\int_0^3 \int_0^{x/3} e^{x^2} dy\, dx = \int_0^3 y e^{x^2}\Big|_0^{x/3} dx$$

$$= \int_0^3 \frac{x}{3} e^{x^2} dx = \frac{1}{6} e^{x^2}\Big|_0^3 = \boxed{\frac{1}{6}(e^9-1)}$$

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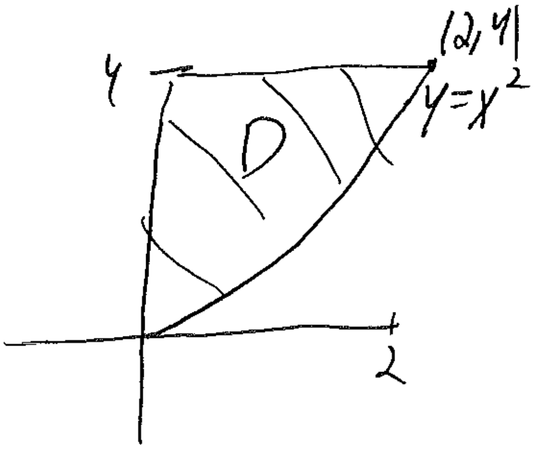
241S3 Quiz #7 - October 29, 2015, 10 a.m.

1. Sketch the region of integration and change the order of integration:

$$\int_0^2 \int_{x^2}^4 f(x,y)\,dy\,dx.$$

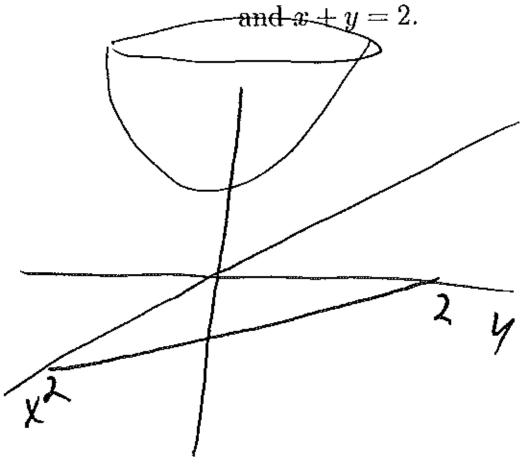


$0 \le y \le 4$

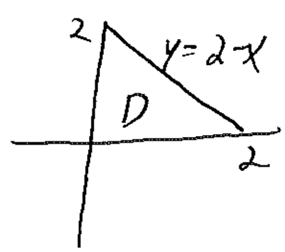
$0 \le x \le \sqrt{y}$

$$\boxed{\int_0^4 \int_0^{\sqrt{y}} f(x,y)\,dx\,dy}$$

2. Find the volume enclosed by the paraboloid $z = x^2 + y^2 + 1$ and the planes $x = 0$, $y = 0$, $z = 0$ and $x + y = 2$.



Want 1^{st} quadrant volume above D, under paraboloid



$0 \le y \le 2-x$

$0 \le x \le 2$

$$\int_0^2 \int_0^{2-x} x^2+y^2+1\,dy\,dx = \int_0^2 x^2y + \frac{y^3}{3} + y \Big|_{y=0}^{2-x} dx$$

$$= \int_0^2 2x^2 - x^3 + \frac{(2-x)^3}{3} + 2 - x\,dx$$

$$= \frac{2}{3}x^3 - \frac{1}{4}x^4 - \frac{(2-x)^4}{12} + 2x - \frac{x^2}{2}\Big|_0^2 = \left(\frac{16}{3} - \frac{16}{4} + 4 - 2\right) - \left(-{}^{16}/_{12}\right)$$

$$= 14/3$$

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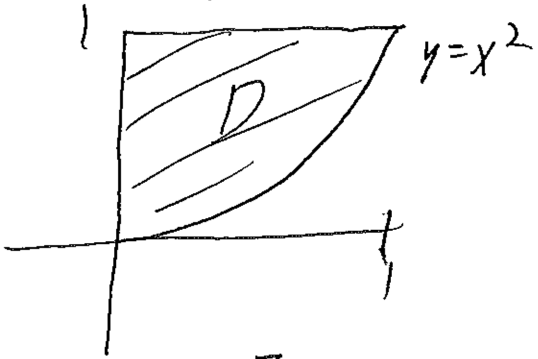
241S2 Quiz #7 - October 29, 2015, 11 a.m.

1. Evaluate

$$\int_0^1 \int_{x^2}^1 \sqrt{y}\sin y \, dy dx$$

by reversing the order of integration.



rewrite as $0 \le y \le 1$
$0 \le x \le \sqrt{y}$

$$\int_0^1 \int_0^{\sqrt{y}} \sqrt{y}\sin y \, dy\, dx = \int_0^1 x\sqrt{y}\sin y \Big|_{x=0}^{x=\sqrt{y}} dx$$

$$= \int_0^1 y \sin y \, dy = -y\cos y + \sin y \Big|_0^1 = -\cos 1 + \sin 1 - 0$$

$$= \boxed{\sin 1 - \cos 1}$$

2. Sketch the region in the x-y plane given in polar coordinates by:

$$1 \le r \le 3, \ 0 \le \theta < \pi/4.$$