

Name:

SOLUTIONS

241S1 Quiz #3 - September 22, 2015, 10 a.m.

1. Let $\vec{r}(t) = (t, 2 \cos t, 2 \sin t)$, $-4 \leq t \leq 4$. Find the length of the curve.

$$\vec{r}'(t) = (1, -2 \sin t, 2 \cos t) \quad |\vec{r}'(t)| = \sqrt{1 + 4 \sin^2 t + 4 \cos^2 t} = \sqrt{5}$$

$$A.L. = \int_{-4}^4 \sqrt{5} dt = 8\sqrt{5}$$

2. Let $\vec{r}(t) = (t, t^2, e^t)$. Find an equation for the unit tangent vector $\vec{T}(t)$.

$$\vec{r}'(t) = (1, 2t, e^t)$$
$$|\vec{r}'(t)| = \sqrt{1 + 4t^2 + e^{2t}}$$

$$\vec{T}(t) = \frac{1}{\sqrt{1 + 4t^2 + e^{2t}}} (1, 2t, e^t)$$

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SOLUTION

241S2 Quiz #3 - September 24, 2015, 11 a.m.

1. The position function of a particle is given by $\vec{r}(t) = (t^2, 5t, t^2 - 2t)$. When is the speed a minimum (Hint: minimize the square)?

$$\vec{r}'(t) = (2t, 5, 2t-2) \quad |\vec{r}'(t)| = \sqrt{4t^2 + 25 + 4t^2 - 8t + 4}$$

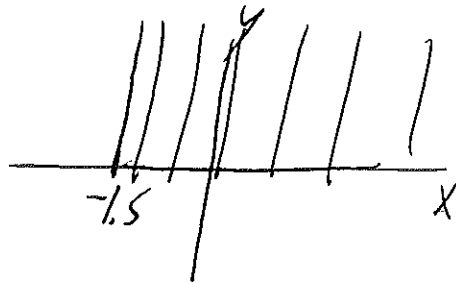
so $(\text{speed})^2 = 8t^2 - 8t + 29$, set derivative equal to zero

$$16t - 8 = 0$$

$$t = \frac{1}{2}$$

2. Find and sketch the domain of the function $f(x, y) = \sqrt{2x+3} + \sqrt{y}$.

Need $y \geq 0$ and $2x+3 \geq 0$ so $x \geq -1.5$



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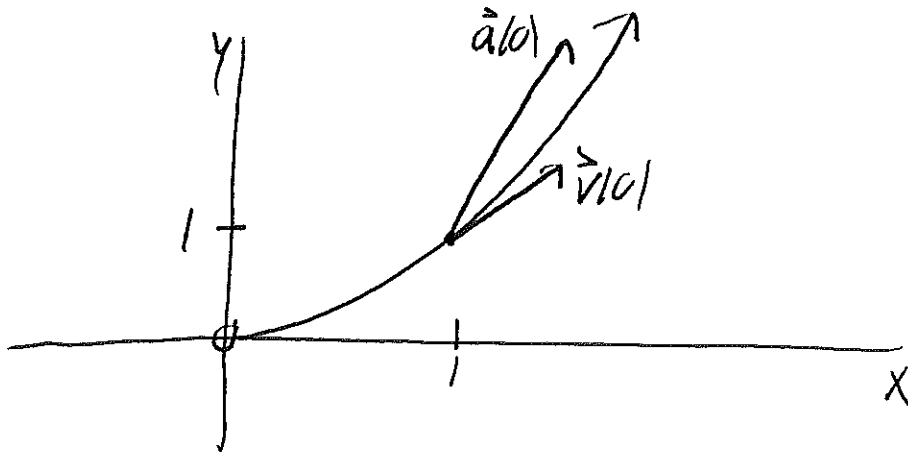
241S3 Quiz #3 - September 24, 2015, 10 a.m.

1. Let $\vec{r}(t) = (e^t, e^{2t})$. Find the velocity, acceleration and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for $t = 0$.

$$\vec{v}(t) = (e^t, 2e^{2t}) \quad \text{speed} = |\vec{v}(t)| = \sqrt{e^{2t} + 4e^{4t}}$$

$$\vec{a}(t) = (e^t, 4e^{2t})$$

Note $\vec{r}(t)$ lies on $y = x^2$ and only for $x > 0$



$$\begin{aligned} \vec{v}(1) &= (e, 2e^2) \\ \vec{a}(1) &= (e, 4e^2) \\ \vec{v}(1) &= (1, 2) \\ \vec{a}(1) &= (1, 4) \end{aligned}$$

2. Let $f(x, y) = y - 2x^2$. Sketch and neatly label the level curves for $k = 0, 1, 4$.

$$k=0 \quad y = 2x^2$$

$$k=1 \quad y = 2x^2 + 1$$

$$k=4 \quad y = 2x^2 + 4$$

