

# SOLUTIONS

Name:

241S1 Quiz #1 - September 8, 2015, 10 a.m.

1. Find the center and radius of the sphere  $x^2 + y^2 + z^2 - 4x + 6y - z = 9$ .

$$x^2 - 4x + 4 + y^2 + 6y + 9 + z^2 - z + \frac{1}{4} = 9 + 4 + 9 + \frac{1}{4}$$
$$(x-2)^2 + (y+3)^2 + (z-\frac{1}{2})^2 = 22.25$$

center  $(2, -3, 1/2)$   
radius  $\sqrt{22.25}$

2. Find the angle between the vectors  $\vec{a} = (2, -1, 3)$  and  $\vec{b} = (1, 1, 5)$ .

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{16}{\sqrt{14} \sqrt{27}}$$

$$\theta = \cos^{-1} \left( \frac{16}{\sqrt{14} \sqrt{27}} \right)$$

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# SOLUTIONS

241S2 Quiz #1 - September 10, 2015, 11 a.m.

1. Let  $\vec{b} = (6, 5, 2)$  and  $\vec{a} = (1, -1, 1)$ . Find the vector projection of  $\vec{b}$  onto  $\vec{a}$ .

$$\frac{\vec{b} \cdot \vec{a}}{\vec{a} \cdot \vec{a}} \cdot \vec{a} = \frac{3}{3} \cdot \vec{a} =$$

$$(1, -1, 1)$$

random  
coincidence  
that this is  
 $\vec{a}$ !

2. Use the scalar triple product to verify that the vectors  $\vec{u} = (1, 5, -2)$ ,  $\vec{v} = (3, -1, 0)$  and  $\vec{w} = (5, 9, -4)$  are coplanar.

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = (1, 5, -2) \cdot (4, 12, 32) = 4 + 60 - 64 = 0$$

scalar triple product is 0  $\leftrightarrow$  coplanar.

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241S2 Quiz #1 - September 10, 2015, 11 a.m.

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2. Use the scalar triple product to verify that the vectors  $\vec{u} = (1, 5, -2)$ ,  $\vec{v} = (3, -1, 0)$  and  $\vec{w} = (5, 9, -4)$  are coplanar.

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Solutions

241S3 Quiz #1 - September 10, 2015, 10 a.m.

1. Let  $\vec{b} = (6, 5, 2)$  and  $\vec{a} = (1, -1, 1)$ . Find the scalar projection of  $\vec{b}$  onto  $\vec{a}$ .

$$\vec{b} \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

2. Find the volume of the parallelepiped with adjacent edges  $PQ$ ,  $PR$  and  $PS$ , where the points are  $P = (-2, 1, 0)$ ,  $Q = (2, 3, 2)$ ,  $R = (1, 4, -1)$ ,  $S = (3, 6, 1)$ .

$$\vec{PQ} = (4, 2, 2) \quad \vec{PR} = (3, 3, -1) \quad \vec{PS} = (5, 5, 1)$$

$$\vec{PQ} \cdot (\vec{PR} \times \vec{PS}) = \vec{PQ} \cdot (8, -8, 0) = 16$$

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241S3 Quiz #1 - September 10, 2015, 10 a.m.

1. Let  $\vec{b} = (6, 5, 2)$  and  $\vec{a} = (1, -1, 1)$ . Find the scalar projection of  $\vec{b}$  onto  $\vec{a}$ .

2. Find the volume of the parallelepiped with adjacent edges  $PQ$ ,  $PR$  and  $PS$ , where the points are  $P = (-2, 1, 0)$ ,  $Q = (2, 3, 2)$ ,  $R = (1, 4, -1)$ ,  $S = (3, 6, 1)$ .