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241S1 Quiz #10 - December 1, 2015, 10 a.m.

1. Let C be the rectangle with vertices $(0,0),(5,0),(5,4),(0,4)$, traversed counterclockwise. Use Green's theorem to evaluate:

$$\oint_C y^2\,dx + x^2 y\,dy.$$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2xy - 2y$ so by G. Thm we want

$$\int_0^5 \int_0^4 2xy-2y \; dy\, dx$$

$$= \int_0^5 xy^2 - y^2 \Big|_0^4 dx = \int_0^5 16x - 16\, dx$$

$$= 8x^2 - 16x \Big|_0^5$$

$$= 200 - 80$$

$$= 120$$

2. Let $\boldsymbol{F}(x,y,z) = (xyz, x^2+yz, y^3+x+z)$. Calculate the curl and the divergence of $\boldsymbol{F}$.

curl $F = (3y^2 - y,\ xy - 1,\ 2x - xz)$

div $F = yz + z + 1$

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241S3 Quiz #10 - December 3, 2015, 10 a.m.

1. Let $\boldsymbol{F}(x, y, z) = (y^2z^3, 2xyz^3, 3xy^2z^2)$. Show that $\boldsymbol{F}$ is conservative by finding a potential function $f(x, y, z)$.

$f(x,y,z) = xy^2z^3$ has $\nabla f = F$.

2. Let $\boldsymbol{r}(u, v) = (u^2, 2u\sin(v), u\cos(v))$ be a parametric surface. Find the equation of the tangent plane to the surface at the point where $u = 1$, $v = 0$.

$\vec{r}_u = (2u, 2\sin v, \cos v)$ $\quad \vec{r}_u(1,0) = (2,0,1)$

$\vec{r}_v = (0, 2u\cos v, -u\sin v)$ $\quad \vec{r}_v(1,0) = (0,2,0)$

$\vec{r}(1,0) = (1,0,1)$

$\boxed{\text{Parametric eq: } (1,0,1) + s(2,0,1) + t(0,2,0)}$

OR

$\vec{n} = (2,0,1) \times (0,2,0) = (-2,0,4)$

$\boxed{-2x + 4z = 2}$

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241S2 Quiz #10 - December 3, 2015, 11 a.m.

1. Find a parametric representation of the part of the hyperboloid $4x^2 - 4y^2 - z^2 = 4$ that lies in front of the yz plane (i.e. $x \geq 0$). Hint: Solve for x.

$x^2 = 1 + y^2 + \frac{z^2}{4}$ $\quad x = \sqrt{1 + y^2 + z^2/4}$ So

$$\boxed{\vec{r}(u,v) = \left(\sqrt{1+u^2+v^2/4},\ u,\ v\right)}$$

2. Let $r(u,v) = (u^2 + 1, v^3 + 1, uv)$ be a parametric surface. Find the equation (in any form you like) of the tangent plane to the surface at the point $(2, 9, 2)$.

$\vec{r}_u = (2u, 0, v)$

point $(2,9,2)$ is when $u=1, v=2$

$\vec{r}_v = (0, 3v^2, u)$

$\vec{r}_u(1,2) = (2,0,2)$ $\quad \vec{r}_v(1,2) = (0,12,1)$

$r_u \times r_v = (-24, -2, 24) \Leftarrow$ use as normal

$$\boxed{\text{Parametric eq: } \vec{r}(s,t) = (2,9,2) + s(2,0,2) + t(0,12,1)}$$

or

$\vec{n} = (-24, -2, 24)$ $\quad -24x - 2y + 24z = ?$

$-48 - 18 + 48 = ?$

$? = -18$

$$\boxed{-24x - 2y + 24z = -18}$$