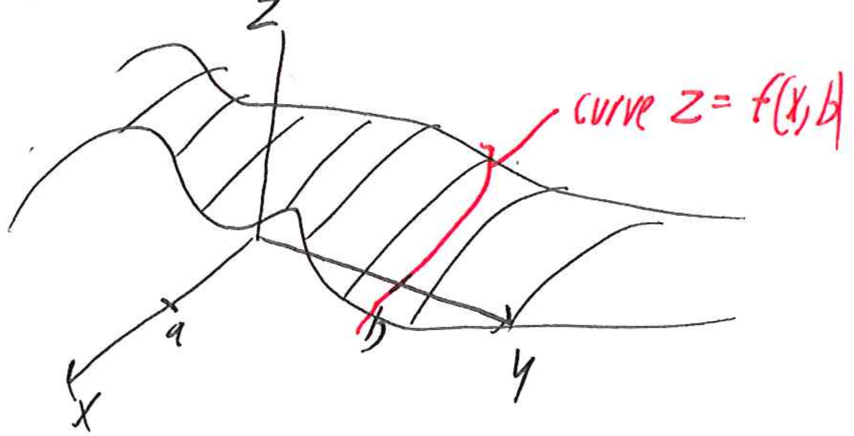
Lecture 9

Review $f(x,y): \mathbb{R}^2 \to \mathbb{R}$, partial derivatives $f_x(x,y) = \lim_{h\to 0} \dfrac{f(x+h,y) - f(x,y)}{h}$

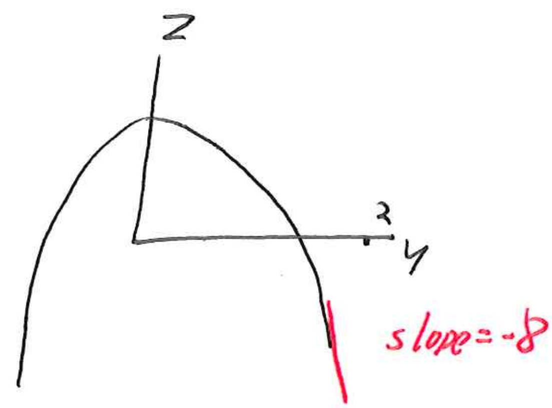
Rmk Setting $y=b$ constant and intersecting plane $y=b$ w/ graph $z=f(x,y)$



Ex $f(x,y) = 4 - x^2 - 2y^2$. Find $f_y(1,2)$ and interpret as a slope

A $f_y = -4y$ $\quad f_y(1,2) = -8$

Setting $x=1$ we get $z = 3 - 2y^2$



Clairaut Thm

$(a,b) \in$ disk $D \subseteq$ domain f. If f_{xy} & f_{yx} are continuous on D

then $f_{xy}(a,b) = f_{yx}(a,b)$

PDE Example: Laplace Equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

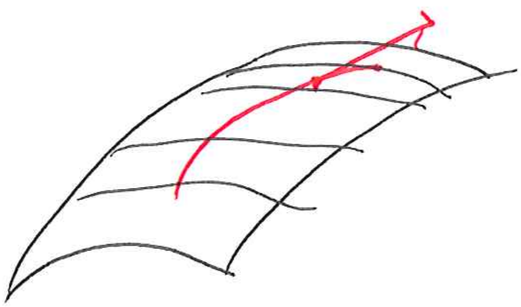
Prove $u(x,y) = e^x \sin y$ is a solution

Ex Show $u(x,t) = \sin(x-at) + \ln(x+at)$ is a solution to the wave equation $u_{tt} = a^2 u_{xx}$.

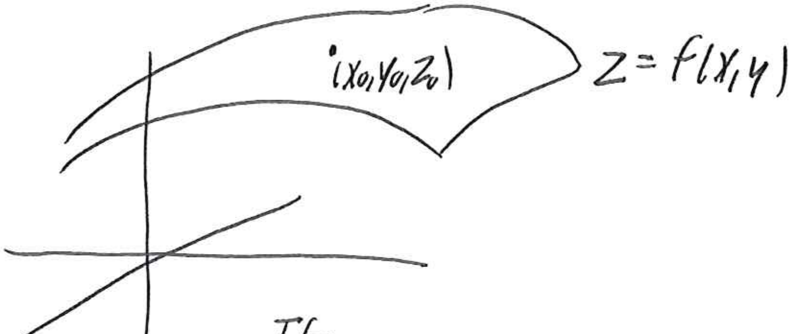
Ex
$e^z = xyz$ Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ by implicit diff.

Ex $z = \ln(x+t^2)$ Find $\partial z/\partial x$, $\frac{\partial z}{\partial t}$ and $\frac{\partial^2 z}{\partial x \partial t}$.

Tangent planes to surfaces



Idea any curve passing through a point has tangent vector parallel to tangent plane at that point.



1. If Tangent plane is not vertical then of form

$$z - z_0 = a(x - x_0) + b(y - y_0)$$

* Set $y = y_0$ gives $z - z_0 = a(x - x_0)$ slope a
* Set $x = x_0$ " $z - z_0 = b(y - y_0)$ slope b

So $a = f_x(x_0, y_0)$ $b = f_y(x_0, y_0)$

Conclude

Thm Suppose $f(x,y)$ has continuous partial derivatives. An equation of the tangent plane to the surface $z = f(x,y)$ at (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Ex $z = \ln(x-2y)$ Find tang plane at $(3,1,0)$

Linear approximations

Note $z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

is eq of tangent plane. Call it $L(x,y)$, linear approx.

Ex Estimate $\sqrt{3.02^2 + 1.97^2 + 5.99^2}$

REVIEW FOR EXAM I