Lecture 8

Review Level curves of $f: \mathbb{R}^2 \to \mathbb{R}$ are curves $f(x,y) = k$, k constant

If $f: \mathbb{R}^3 \to \mathbb{R}$ we get level surfaces $f(x,y,z) = k$.

- useful for graphing.

Ex $f(x,y) = ye^x$

Fancier plots on maple.

$f(x,y,z) = x^2+y^2+z^2$

Limits Compare $f(x,y) = \dfrac{\sin(x^2+y^2)}{x^2+y^2}$ and $g(x,y) = \dfrac{x^2-y^2}{x^2+y^2}$

as $(x,y) \to (0,0)$

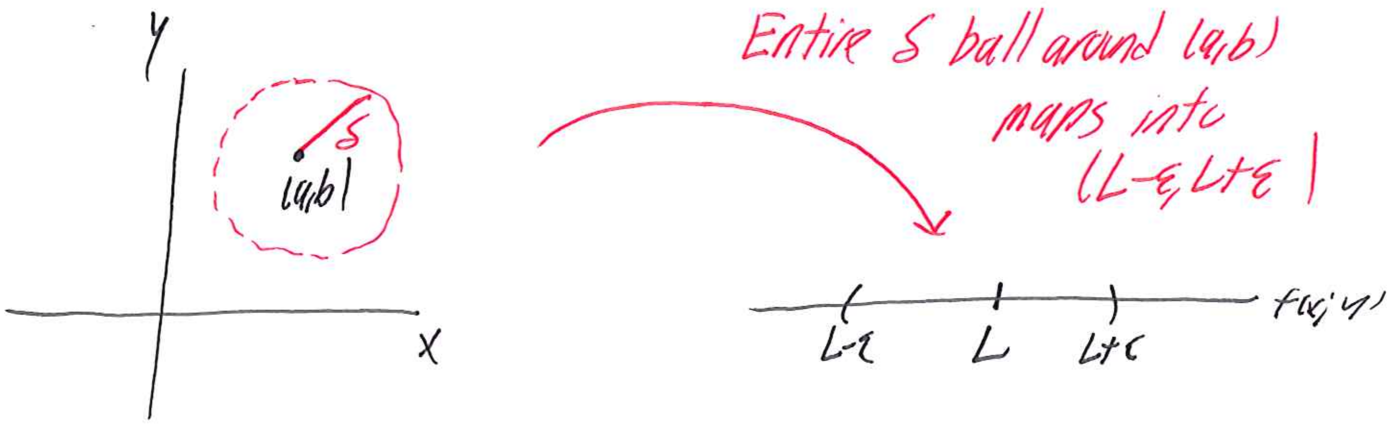
Recall: $\lim_{h \to 0} \frac{\sin h}{h} = 1$

Claim $\lim_{(x,y)\to(0,0)} f(x,y) = 1$ $\lim_{(x,y)\to(0,0)} g(x,y)$ DNE

Def Suppose domain D of $f(x,y)$ contains a small disc around (a,b) (Not including (a,b))

Say $\lim_{(x,y)\to(a,b)} f(x) = L$ if for every $\varepsilon > 0$ $\exists$ $\delta > 0$ s.t.

if $(x,y) \in D$ and $0 < \sqrt{(x-a)^2+(y-b)^2} < \delta$ then $|f(x,y) - L| < \varepsilon$.



Rmk In calc 1 $x \to a$ from left or right. In multivariable (x,y) can approach (a,b) along any curve!

Ex $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ DNE

Pf On line $y=ax$, $g(x,y) = \dfrac{x^2-a^2x^2}{x^2+a^2x^2} = \dfrac{(1-a^2)x^2}{(1+a^2)x^2} = \dfrac{1-a^2}{1+a^2}$

or on x axis $f=1$, on y axis $f=-1$!

Ex $f(x,y) = \dfrac{xy}{x^2+y^2}$ on axes $f=0$, on line $y=x$ $f=1/2$.
Limit DNE

Ex $f(x,y) = \dfrac{xy^2}{x^2+y^4}$ Does $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

* same limit on any line $y=mx$

* Diffent limit on $x=y^3$

* Proving limits exist is difficult!

Recall $f(x,y,z)$ is continuous at (a,b,c) if

$$\lim_{(x,y,z)\to(a,b,c)} f(x,y,z) = f(a,b,c)$$

Exs polynomials, exponentials, logs, trig, etc..

Ex

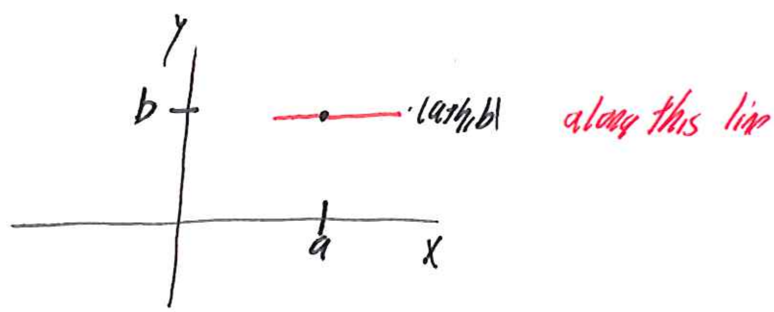
- $\dfrac{xy}{1+e^{x-y}}$ continuous everywhere
- $\ln(1+x-y)$ continuous when $1+x-y>0$
- $\dfrac{e^x+e^y}{e^{xy}-1}$ continuous except if $x=0$ or $y=0$.

14.3 Partial Derivatives

Idea $f(x,y)$ function of >1 variable, hold all but one constant.

Ex

$$f_x(a,b)=\lim_{h\to 0}\frac{f(a+h,b)-f(a,b)}{h}$$



Similarly $f_y(a,b)$ called partial derivative of f w.r.t. x and y.

$$f_x(x,y)=\lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$$

Notation $z=f(x,y)$ then

$$f_x(x,y)=f_x=\frac{\partial f}{\partial x}=\frac{\partial z}{\partial x}=\frac{\partial}{\partial x}f(x,y)=f_1=D_x f$$

How to calculate? Hold other variables as constant

Ex $f(x,y) = x^2 + x\sin y + e^{x^2+y^2}$

$f_x = 2x + \sin y + 2xe^{x^2+y^3}$

$f_y = 0 + x\cos y + 2ye^{x^2+y^2}$

Ex $x^3 + y^3 + z^3 + 6xyz = 1$. Find $\frac{\partial z}{\partial x}$ w/ implicit diff.

$3x^2 + 3z^2\frac{\partial z}{\partial x} + 6yz + 6xy\frac{\partial z}{\partial x} = 0$

$$\frac{\partial z}{\partial x} = \frac{x^2+2yz}{z^2+2xy}$$

Ex $f(x,y,z) = e^{xyz^2}$ Find partial deriv.

Ex $f = x^2 + xy + x^3y^2$ Find $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$

$$\frac{\partial^2 f}{\partial x \partial y} = f_{xy} \qquad \frac{\partial^2 f}{\partial y \partial x} = f_{yx}$$

Thm (Clairaut) Suppose f defined on a disc D containing (a,b) and f_{xy}, f_{yx} continuous on D. ~~Then~~

Then $f_{xy}(a,b) = f_{yx}(a,b)$

= "equality of mixed partials"

Problems 14.3

#4 Estimate $f_v(40,15)$ from table

#55 $Z = \dfrac{y}{2x+3y}$ Find all 2^{nd} partials

#5-8 w/ visualizer

#77 Verify $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of

Laplace Eq $u_{xx} + u_{yy} + u_{zz} = 0$