Lecture 7

Rmk If $\vec{r}(t)$ is position at time t then $\vec{r}'(t) = \vec{v}(t)$ = velocity, $|\vec{r}'(t)|$ = speed
$|\vec{r}''(t)|$ = acceleration

Example Particle has init position $\vec{r}(0) = (1,2,3)$ and init velocity $(-1,1,2)$.
Suppose $\vec{a}(t) = (t, 2-t^2, t^3)$. Find $\vec{r}(t)$.

Newton's 2nd Law Force $\vec{F}(t)$ acts on object of mass m then

$$\vec{F}(t) = m\vec{a}(t)$$

Example Object moves on circle radius a, angular speed ω, so

$$\vec{r}(t) = (a\cos\omega t, a\sin\omega t)$$
$$r'(t) = (-a\omega\sin(\omega t), a\omega\cos\omega t)$$
$$\vec{r}''(t) = (-a\omega^2\cos\omega t, -a\omega^2\sin(\omega t)) \leftarrow \text{Notice this equals } -\omega^2\vec{r}(t)$$

So $\vec{F}(t) = m\vec{a}(t) = \underline{-m\omega^2\vec{r}(t)}$

Centripetal Force - points back toward $\vec{0}$

#28 Batter hits baseball 3ft above ground toward fence 10 ft high and 400 ft away.

Ball leaves bat at 115 ft/s at $\measuredangle$ 50° above horizontal.

Is it a home run?

Put origin at feet so $\vec{r}(0)=(0,3)$, $\vec{v}(0)=(115\cos 50, 115\sin 50)\approx(73.92, 88.10)$

$\vec{a}(t)=(0,-32)$ so $\vec{v}(t)=(73.92, 88.10-32t)$ $\vec{r}(t)=(73.92t, -16t^2+88.1t+3)$

Gets to wall when $400=73.92t$ so $t=5.41$

$\vec{r}(5.41)=(400, 11.33)$ Home run!

13.426 Tank fires 400 m/s. What two angles of elev to hit target 3000 meters away

$\vec{a}(t)=(0,-9.8\ m/sec^2)$ $\vec{v}(0)=(400\cos\theta, 400\sin\theta)$ $\vec{r}(0)=(0,0)$

$\vec{v}(t)=(400\cos\theta, -9.8t+400\sin\theta)$

$\vec{r}(t)=(400\cos\theta\, t, -4.9t^2+400\sin\theta\, t)$

Hits ground when $-4.9t^2+400\sin\theta\, t=0$ so $t=0$ or $t=\frac{400\sin\theta}{4.9}$

Want $(400\cos\theta)\left(\frac{400}{4.9}\sin\theta\right)=3000$

$32653\underbrace{\sin\theta\cos\theta}_{\frac{1}{2}\sin 2\theta}=3000$

$\sin 2\theta=.18375$ $2\theta=\sin^{-1}(.18375)$

$=10.58$ or 169.42

$\boxed{\theta=5.3^\circ \text{ or } 84.7^\circ}$

Recall $\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} \Rightarrow \vec{v}(t) = v\vec{T}$

$\vec{a} = \vec{v}' = v'\vec{T} + v\vec{T}'$ recall $\kappa = \frac{|T'|}{v}$ so $|T'| = \kappa v$

$\vec{N} = T'/|T'|$

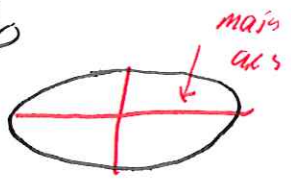
$\vec{a} = v'\vec{T} + \kappa v^2 \vec{N}$

Resolves acceleration into tangential and normal components

$\vec{a} = a_T \vec{T} + a_N \vec{N}$

Highly Recommend reading 13.4 Kepler's Laws

1. Planet orbits sun on ellipse w/ sun at focus
2. Equal areas = times
3. Square of period of revolution proportional to cube of length of major axis



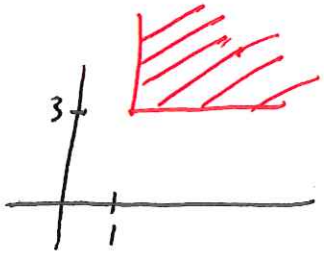
Chapter 14

Now we consider real-valued functions of more than one variable:

Ex $f(x,y,z) = x^2 \sin z - xy^3$ $\quad f: \mathbb{R}^3 \to \mathbb{R}$

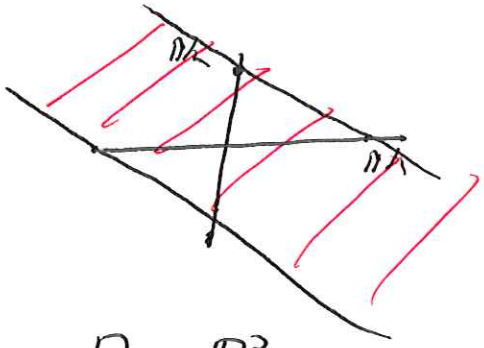
i.e. domain is some subset of $\mathbb{R}^n$

Ex $f(x,y) = \sqrt{x-1} + x\sqrt{y-3}$ Domain



Ex $f(x+y) = \sin^{-1}(x+y)$

$D: \; -\pi/2 \le x+y \le \pi/2$

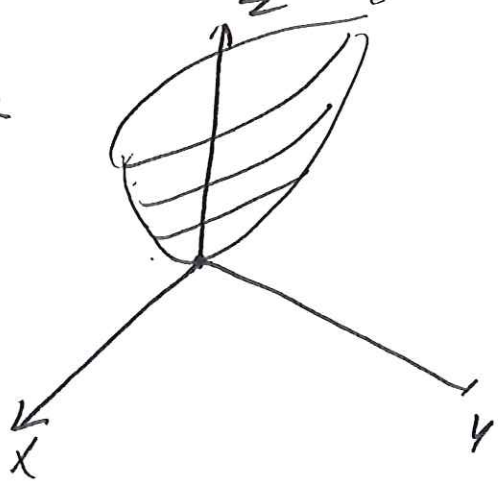


Graphs Suppose $f(x,y)$ has domain $D \subseteq \mathbb{R}^2$.

Graph is all points $\{(x,y,f(x,y)) \mid (x,y) \in D\}$.

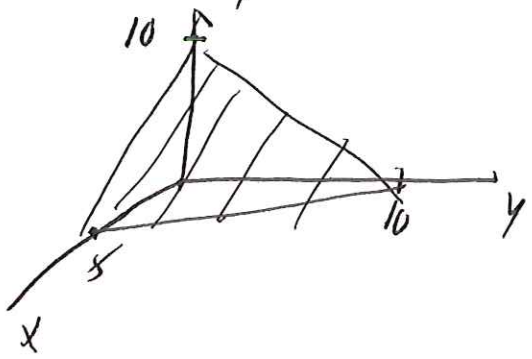
usually a surface

Ex $f(x,y) = x^2 + 2y^2$



traces z = constant are ellipses longer in x direction

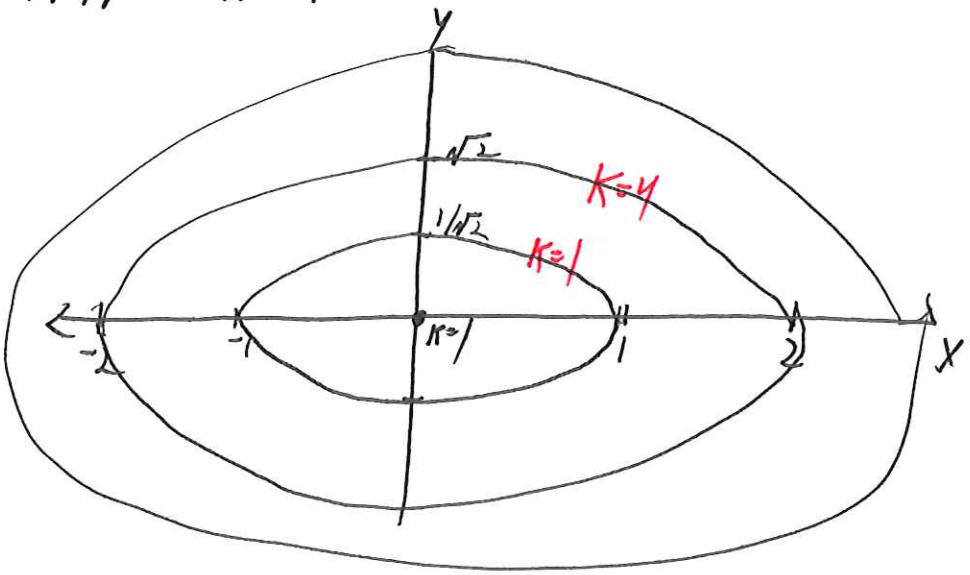
EX $f(x,y) = 10 - 2x - y$ Graph $z = 10 - 2x - y$ is a plane



Level Curves

Def The level curves of $f(x,y)$ are curves $f(x,y) = k$

EX $f(x,y) = x^2 + 2y^2$



etc.

EX Given $f(x,y,z)$ you can sketch level surfaces in $\mathbb{R}^3$?

Problems

14.1 #32 (visualize), #45, #48,