Lecture 5

Review $\vec{r}(t) = \vec{r}_0 + t\vec{a}$ param. a line, Ex $\vec{r}(t) = \langle -1, 1, 2\rangle + t\langle 3, 3, 6\rangle$

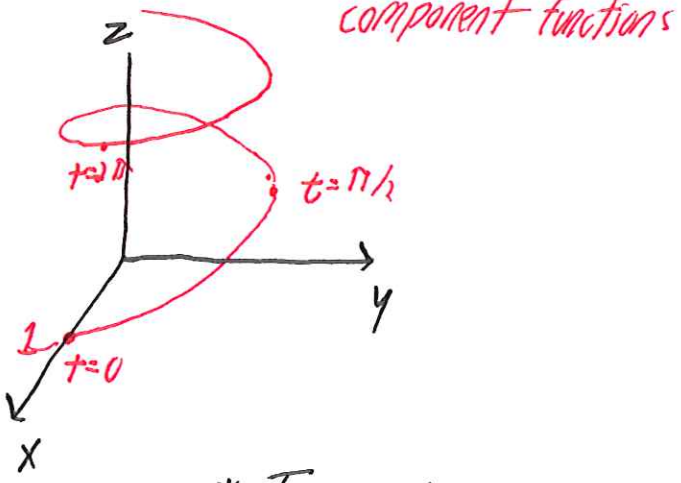
Rmk $\vec{r}(t)$ is a <u>vector valued function</u> of a real variable: $\vec{r}: \mathbb{R} \to \mathbb{R}^3$

Space Curves

Ex $\vec{r}(t) = \langle \cos t, \sin t\rangle$ traces out a circle of radius 1

$\vec{r}(t) = \langle \cos t, \sin t, t\rangle$ curves spirals up cylinder $x^2 + y^2 = 1$

component functions



Parametric equations:

$x = \cos t$

$y = \sin t$

$z = t$

* Image is a space curve

* Parametrization contains info about time

Ex $\vec{r}(t) = \langle \cos 2t, \sin 2t, 2t\rangle$ travels twice as fast.

Problem Find a parametrization of curve of intersection of $x^2 + y^2 = 4$ and $z = xy$

A: $\vec{r}(t) = \langle 2\cos t, 2\sin t, 4\cos t \sin t\rangle$

Prob Two particles travel on space curves

$\vec{r}_1(t) = \langle t, t^2, t^3 \rangle \quad \vec{r}_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle$

- Do they collide?
- Do paths intersect?

Prob p. 854 #21-26 on Visualizer

Rmk Complicated curves easy to plot using computer

Ex $\vec{r}(t) = (t^3 - 3t, t^4 - 4t^2, t^5 - 10t)$ trefoil (Shastri 1992)

Ex Mathematica 3-4 torus knot

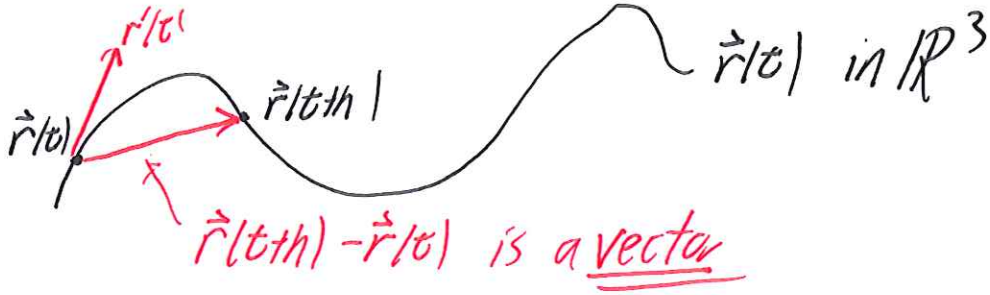
Open Problem Given a knot, what is smallest degree of polynomials needed to parametrize a curve realizing the knot?

Calculus Begins

Def Suppose $\vec{r}(t) = (f(t), g(t), h(t))$. Then $\lim_{t \to a} \vec{r}(t) = (\lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t))$ if the limits of component functions exist

Def $\vec{r}(t)$ is <u>continuous at a</u> if $\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$

Derivatives



Def $\vec{r}\,'(t) = \lim_{h \to 0} \dfrac{\vec{r}(t+h) - \vec{r}(t)}{h}$

Easy Fact If $\vec{r}(t) = (f(t), g(t), h(t))$ then $\vec{r}'(t) = (f'(t), g'(t), h'(t))$

Def $\vec{r}'(t)$ is called tangent vector

The tangent line through a point P on curve is // to tan vect

Ex Find parametric equation of the tangent line to

$$\vec{r}(t) = (t^2+1, 4\sqrt{t}, e^{t^2-t}) \text{ at } (2,4,1)$$

Ex Let $\vec{r}(t) = (\sqrt{t-2}, 3, 1/t^2)$. Find unit tangent vector at time $t=3$.

Ex $\vec{r}(t) = (\cos t, \sin t)$

Rules Assume $\vec{u}(t)$, $\vec{v}(t)$ are differentiable vector functions, c is a scalar and $f(t)$ a real valued function.

1. $\frac{d}{dt}(\vec{u}(t) + \vec{v}(t)) = \vec{u}'(t) + \vec{v}'(t)$
2. $\frac{d}{dt}(c\vec{u}(t)) = c\vec{u}'(t)$
3. $\frac{d}{dt}(f(t)\vec{u}(t)) = f'(t)\vec{u}(t) + f(t)\vec{u}'(t)$
4. $\frac{d}{dt}(\vec{u}(t) \cdot \vec{v}(t)) = \vec{u}'(t) \cdot \vec{v}(t) + \vec{u}(t) \cdot \vec{v}'(t)$
5. $\frac{d}{dt}(\vec{u}(t) \times \vec{v}(t)) = \vec{u}'(t) \times \vec{v}(t) + \vec{u}(t) \times \vec{v}'(t)$
6. $\frac{d}{dt}\vec{u}(f(t)) = \vec{u}'(f(t)) \cdot f'(t)$

Examples

#11 $\vec{r}(t) = (t^2, \cos(t^2), \sin(t^2))$ Find $\vec{r}'(t)$

Suppose $|\vec{r}(t)|$ is constant Prove $\vec{r}'(t) \perp$ to $\vec{r}(t)$

Integrals

Fact Can calculate $\int_a^b \vec{r}(t)\,dt$ coordinate wise

Ex $\int \vec{r}(t)\,dt$ for $\vec{r}(t) = (t, t^2, t^3)$

Ex $\int_0^2 t\vec{i} - t^2\vec{k} + t^3\vec{j}\,dt$

Problems 13.2 #6
#16
#22
#28