Lecture 4

Review
- Cross product $\vec{a} \times \vec{b}$ is $\perp$ to $\vec{a}$ and $\vec{b}$
- $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$, $\theta = \measuredangle$ btw $\vec{a}$ & $\vec{b}$, $\vec{a} \times \vec{b} = 0 \leftrightarrow \vec{a}, \vec{b}$ parallel
  = area of parallelogram
- direction of $\vec{a} \times \vec{b}$ given by rt hand rule
- Volume of parallelepiped is $|\,|$ of scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$
- Torque $\tau = \vec{r} \times \vec{F}$
  position vectr for ↗ ↖ force

Lines and Planes

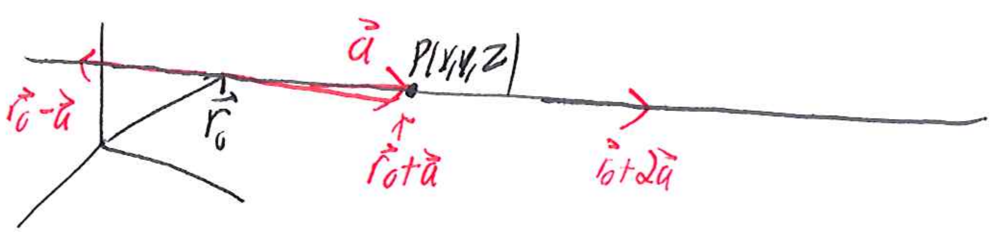
Problem In $\mathbb{R}^2$ $\{ax+by=c\}$ gives all possible lines but in $\mathbb{R}^3$

$ax+by+cz=d$ is a plane. How to write eq of line in dimension ≥ 3?

A1 As intersection of two planes. This is awkward!

Better Answer Parametrically.

Idea Pick a point (x_0, y_0, z_0) on line, pos. vector $\vec{r}_0$



* Observe for any point on the line, the position vectors from $\vec{r}_0$ to P are all parallel

* The entire line is all the points $\vec{r}_0 + t\vec{v}$ as $t \in \mathbb{R}$

Conclude $\vec{r} = \vec{r}_0 + t\vec{v}$ is vector equation of a line $\left(\begin{array}{c}\text{works in} \\ \text{any dimension!}\end{array}\right)$ Q

Ex $\vec{r}(t) = (3,-2,6) + t(1,2,3)$

Q Is $(1,-6,0)$ on this line?

$\rightarrow (x,y,z) = (3+t, -2+2t, 6+3t)$

Parametric equations
$$x = 3+t$$
$$y = -2+2t$$
$$z = 6+3t$$

* easy to go back & forth vector $\leftrightarrow$ parametric

Ex Find vector equation of line through $(2,-1,6)$ and $\parallel$ to $\vec{r}(t) = (1,1,1) + t(-1,2,1)$

Rmk Solving each parametric equation for t gives symmetric eq

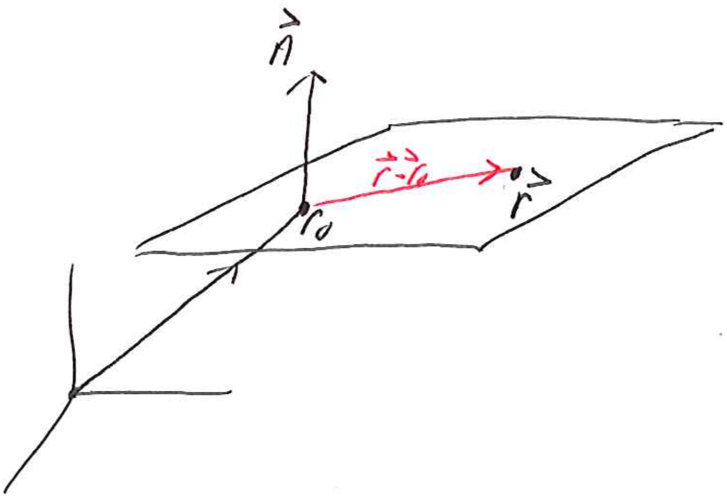
Above Ex
$$\frac{x-3}{1} = \frac{y+2}{2} = \frac{z-6}{3}$$

Difficulty Each line has infinitely many parametrizations!

Ex
$$\vec{r}_1(t) = (2,3,4) + t(6,0,1)$$
$$\vec{r}_2(t) = (-10,3,2) + t(12,0,2)$$
same line!

Planes in $\mathbb{R}^3$

Rmk A plane in $\mathbb{R}^3$ is entirely determined by a point and a normal vector $\vec{n}$ perpendicular to the plane.



$\vec{r}$ is in the plane if & only if $(\vec{r}-\vec{r_0}) \perp \vec{n}$,
i.e. $(\vec{r}-\vec{r_0}) \cdot \vec{n} = 0$.

Vector Equation of Plane $\quad \vec{n}\cdot(\vec{r}-\vec{r_0})=0 \quad$ where $\vec{r}=(x,y,z)$
$\vec{r_0}$ is a point on plane

Ex $\vec{n}=(1,2,3)$
$\vec{r_0}=(4,5,8)$

$$(1,2,3)\cdot(x-4,y-5,z-8)=0$$
$$x-4+2y-10+3z-8=0$$
$$\left.\begin{array}{c} x+2y+3z=22 \\ \text{or} \\ 1\cdot(x-4)+2\cdot(y-5)+3\cdot(z-8) \end{array}\right\} \text{scalar equation}$$

Note: can read off normal vector

Problems

1. Find eq of plane through $(2,1,6)$ and $\parallel$ to $x-3y+2z=1$

2. Find vector eq of plane through points $(1,2,0)$, $(-1,2,1)$, $(1,1,1)$

3a. Find $\measuredangle$ btw planes $x+y+z=1$ and $x-y+2z=2$

b. Find parametric & symmetric eq for line of intersection

4. Find point where lines $\vec{r}=(1,1,0)+t(1,-1,2)$
$\vec{r}=(2,0,2)+s(-1,1,0)$ intersect

Find eq of plane containing them

5. Parametrize a line segment from $(1,2,3,4)$
to $(5,5,5,5)$ in $\mathbb{R}^4$

6. Where does line through $(-3,1,0)$ and $(-1,5,6)$ intersect
plane $2x+y-z=-2$.

12.6 Cylinders and Quadrics

<u>Define</u> Cylinder = all lines // to a given and passing through a plane curve

<u>Ex</u> $x = z^2$

$y = \sin x$

$x^2 + y^2 = 1$

Quadric Surfaces

Any equation in x, y, z of degree 2.

<u>Fact</u> Translate and rotate to get

$$Ax^2 + By^2 + Cz^2 + J = 0 \quad \text{or}$$

$$Ax^2 + By^2 + Iz = 0$$

<u>Idea</u> Set x, y, and/or $z = 0$ to sketch

- $x^2 + 4y^2 + 9z^2 = 0$
- $z = 4x^2 + y^2$
- $z = y^2 - x^2$ hyperbolic paraboloid = saddle

Review p. 837