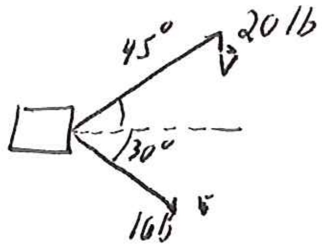
Lecture 2

Review

- add vectors
- $c\vec{v}$ meaning depends on $c>0$ or $c<0$
- Coordinates $\langle a_1, a_2, a_3 \rangle$ is vector w/ tail at origin and tip at (a_1, a_2, a_3)

* vector operations can be done coord wise.

Ex



$$\vec{v} = (20\cos 45, 20\sin 45) = (10\sqrt{2}, 10\sqrt{2})$$

$$\vec{w} = (16\cos 30, -16\sin 30) = (8\sqrt{3}, -8)$$

$$\vec{v} + \vec{w} = (10\sqrt{2} + 8\sqrt{3}, 10\sqrt{2} - 8) \approx (28, 6.14)$$

Fact The **length** or **magnitude** of $\vec{v} = (a_1, a_2, a_3)$ is $|\vec{v}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Ex Find a unit vector in same direction as $\vec{v} = (2, -1, 5)$

Standard Basis Vectors $\vec{i} = (1,0,0)$ $\vec{j} = (0,1,0)$ $\vec{k} = (0,0,1)$

Ex $\vec{v}$ above $= 10\sqrt{2}\,\vec{i} + 10\sqrt{2}\,\vec{j}$

Rmk Basic Properties of vector ops in $\mathbb{R}^2$

Ex $\vec{a} + -\vec{a} = \vec{0}$ etc.

Dot products

Def $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$ $\quad \vec{a}\cdot\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

similarly for other dim.

aka scalar product $\qquad (3,-1,2)\cdot(1,1,6) = 3 - 1 + 12 = 14$

Properties

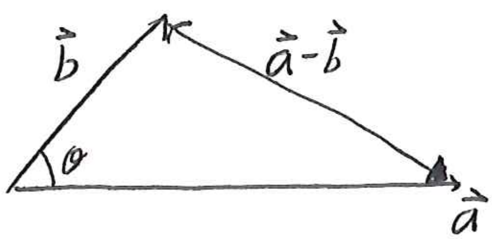
1. $\vec{a}\cdot\vec{a} = |\vec{a}|^2$ $\quad$ 2. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ $\quad$ 3. $\vec{a}\cdot(\vec{b}+\vec{c})$

4. $(c\vec{a})\cdot\vec{b} = ..$ $\quad$ 5. $\vec{0}\cdot\vec{a} = 0$

Key Formula $\quad \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta \qquad \theta$ is $\measuredangle$ btw

- This can be the defn of $\cdot$
- In higher dims can def $\measuredangle$ this w.

Proof



Law of Cosines $\quad |\vec{a}-\vec{b}|^2 = \cancel{|\vec{a}|+|\vec{b}|}\ |\vec{a}|^2 + |\vec{b}|^2 - 2|a||b|\cos\theta$

$(a-b)\cdot(a-b) = a\cdot a + b\,b - 2|a||b|\cos\theta$

$-2a\cdot b = -2|a||b|\cos\theta \qquad //$

Cor $\quad \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

Cor $\quad \vec{a}$ & $\vec{b}$ are orthogonal $\Leftrightarrow \vec{a}\cdot\vec{b} = 0$

Ex Find $\measuredangle$ btw $2\vec{i}+3\vec{j}+\vec{k}$ and $\vec{i}-\vec{j}-\vec{k}$

$$(2,3,1)\cdot(1,-1,-1) = -2$$

$$\cos\theta = \frac{-2}{\sqrt{14}\sqrt{3}} \approx -.308 \qquad \theta = 1.88 \approx 107^\circ$$

Direction $\measuredangle$ $\vec{a}$ vector $\alpha = \measuredangle$ w/ x axis: $\dfrac{a\cdot\vec{i}}{|a|} = \dfrac{a_1}{|a|}$

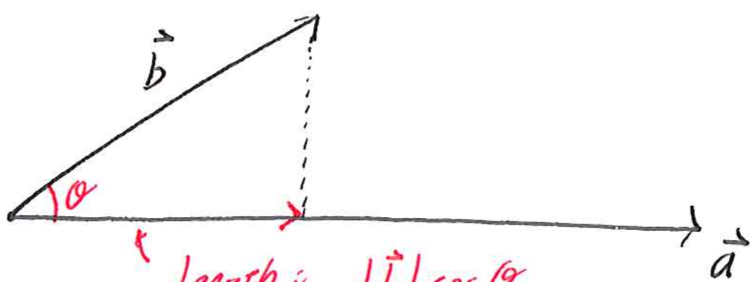
β

γ

$\cos\alpha, \cos\beta, \cos\gamma$ is a unit vec $\dfrac{\vec{a}}{|\vec{a}|} = (\cos\alpha, \cos\beta, \cos\gamma)$

Projections

Motivation Want component of one vector in dir of anot.



length is $|\vec{b}|\cos\theta$

$= |\vec{b}| \cdot \dfrac{\vec{a}\cdot\vec{b}}{|a|\cdot|b|}$ vector is $\dfrac{\vec{a}}{|\vec{a}|}$

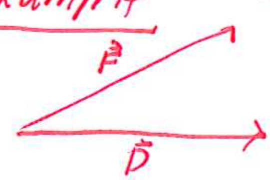
Conclude

Vector projection of $\vec{b}$ onto $\vec{a}$ is

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\cdot\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\cdot\vec{a}$$

Scalar projection $\text{comp}_a b = \dfrac{a\cdot b}{|\vec{a}|} = \vec{b}\cdot\left(\dfrac{\vec{a}}{|\vec{a}|}\right)$

Example 1. Scal & Vect proj of $\vec{b} = (1,-1,3)$ onto $\vec{a} = (1,1,$

2\. 

Force $\vec{F}$, Disp $\vec{D}$

$W = |\vec{F}|\cos\theta|\vec{D}| = \vec{F}\cdot\vec{D}$

Problems

1. Find a unit vector $\perp$ to $(1,1,2)$ and $(-1,1,0)$

2. Find $\measuredangle$ btw $y=x^2$ & $y=x^3$ at pts of $\cap$

3. $\vec{F}=8\vec{i}-6\vec{j}+9\vec{k}$ moves object from $(0,10,2)$ to $(6,12,20)$ d in meters F in Newtons

   Find Work

4. $(2,0)$, $(0,3)$ $(3,4)$

   Find $\measuredangle$s in Δ