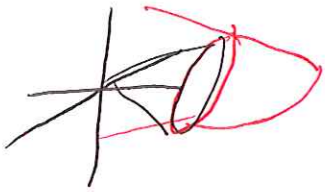
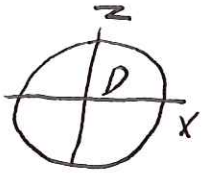
Lecture 18

Ex Find volume enclosed by $y = x^2 + z^2$ & $y = 8 - x^2 - z^2$



Intersection at $x^2 + z^2 = 4$



$$\iint_D \int_{x^2+z^2}^{8-x^2-z^2} 1 \, dy$$

$$= \iint_D 8 - 2x^2 - 2z^2 \, dA \quad \leftarrow \text{now switch to polar!}$$

Ex $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\,dz\,dz$. Sketch the solid whose volume is given by this $\iiint$

Ex . Find region E that maximizes $\iiint_E 1 - x^2 - 2y^2 - 3z^2 \, dV$
- use comp alg to find max!

Ex $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\, dz\,dy\,dx$. Rewrite in 5 other ways.

Cylindrical Coordinates

- Polar in 2-dimensions and 3rd as usual

Ex
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

Ex Sketch $r = 2$
Sketch $\theta = \pi/4$

Sketch $z = r$, $z = r^2$

Triple integrals in cylindrical coor.

- As before just $\iiint_E \_\_ dz\,dx\,dy$ — change to polar

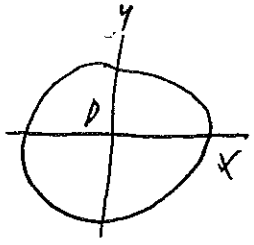
Note $dx\,dy\,dz \Rightarrow r\,dr\,d\theta\,dz$

Ex $\int_{-\pi/2}^{\pi/2} \int_0^2 \int_0^{r^2} r\,dz\,dr\,d\theta$ Sketch solid whose vol is given & evaluate

Ex Evaluate $\iiint_E z\,dV$ E enclosed by $z = x^2 + y^2$ and $z = 4$

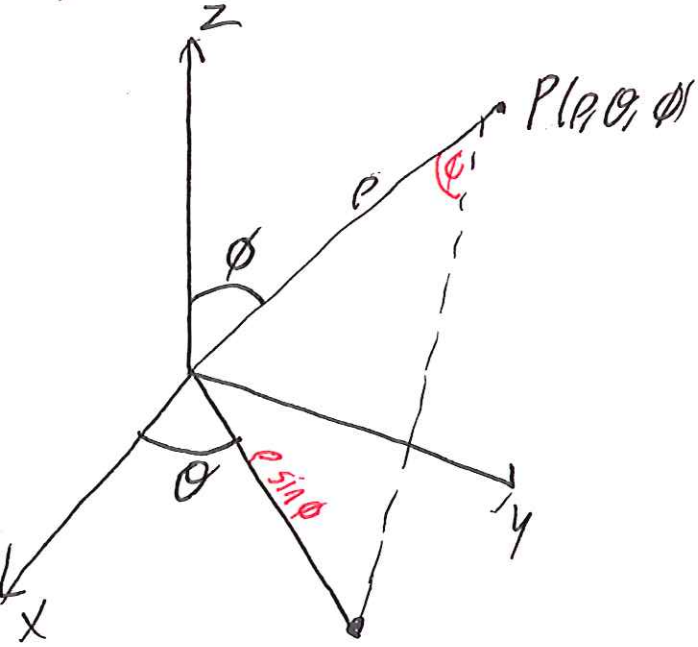
Ex $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy$ evaluate by changing to cyl'rs

$$\int_0^{2\pi} \int_0^2 \int_r^2 r\cos\theta \cdot z \cdot r\,dz\,dr\,d\theta$$



Ex Find volume of solid inside $x^2+y^2=1$ and $x^2+y^2+z^2=4$

Spherical Coord



$0 \le \rho$

$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$

Check

$z = \rho \cos\phi$

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$$\rho = \sqrt{x^2+y^2+z^2}$$

Ex Convert $(1,2,3)$ in rectangular to spherical

$\rho = \sqrt{14}$ $\quad \cos\phi = z/\rho = 3/\sqrt{14}$ $\quad \phi = \cos^{-1}(3/\sqrt{14})$

$\sin\phi = \sqrt{5}/\sqrt{14}$

$\cos\theta = x/\rho\sin\phi = 1/\sqrt{5}$ $\quad \theta = \cos^{-1}(1/\sqrt{5})$

which one ?

Ex Sketch $\rho = 3$

Sketch $\phi = \pi/3$

Sketch $\theta = \pi/4$

Triple Integrals in Spherical

$$\iiint_E f(x,y,z)\,dx\,dy\,dz$$

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$

$$dx\,dy\,dz \Rightarrow \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

EX $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\,dV$ B is unit ball

EX Write $z = x^2+y^2$ in spherical

$$\rho\cos\phi = \rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta$$
$$= \rho^2\sin^2\phi$$

$$\boxed{\cos\phi = \rho\sin^2\phi}$$