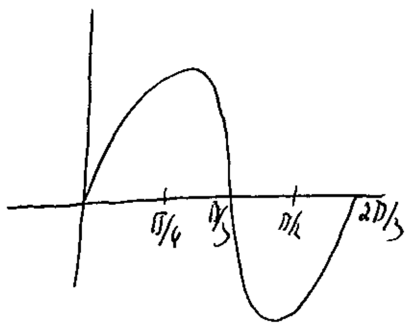
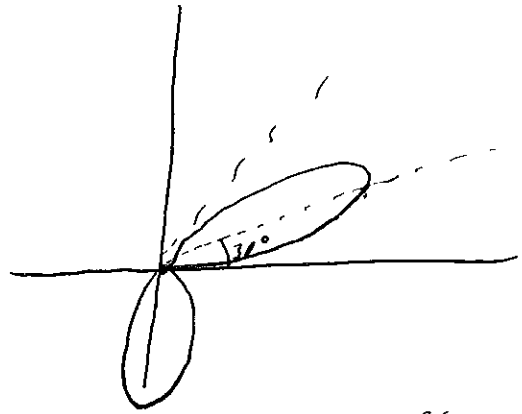
Lecture 17

Problem Find area enclosed by one leaf of $r = \sin(3\theta)$





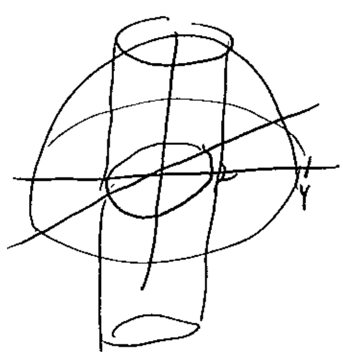
$0 \le \theta \le \pi/3$
$0 \le r \le \sin 3\theta$

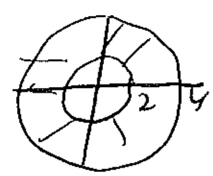
$$A = \int_0^{\pi/3}\int_0^{\sin 3\theta} r\,dr\,d\theta = \int_0^{\pi/3} \frac{r^2}{2}\Big|_0^{\sin 3\theta} d\theta = \int_0^{\pi/3} \frac{\sin^2(3\theta)}{2}\,d\theta$$

$$= -\frac{1}{6}\sin(3\theta)\cos(3\theta) + \frac{1}{2}\theta\Big|_0^{\pi/3}$$

$$= \pi/6$$

Problem Use polar coord to find volume inside $x^2+y^2+z^2=16$ and outside $x^2+y^2=4$



Let $D=$ 

$$V = 2\cdot \iint_D \sqrt{16-x^2-y^2}$$

Switch to polar: $$V = 2\int_0^{2\pi}\int_2^4 \sqrt{16-r^2}\, r\,dr\,d\theta$$

$$= 2\int_0^{2\pi} (16-r^2)^{3/2}\cdot -\frac{1}{3}\Big|_2^4 d\theta$$

$$= 2\int_0^{2\pi} 0 + \frac{1}{3}\cdot 12^{3/2}\,d\theta = \frac{2}{3}\pi\cdot 2\cdot 12^{3/2}$$

Triple Integrals

Once again... $B = [a,b] \times [c,d] \times [r,s]$, split into cubes

$$\iiint_B f(x,y,z)\,dV = \lim_{l,m,n \to \infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_i^*, y_j^*, z_k^*)\,\Delta V$$

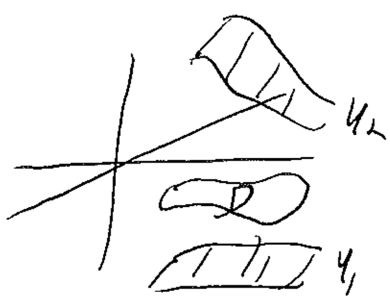
triple integral. = 4-dim'l volumes

Rmk Fubini's Thm still holds

Ex $\int_0^2 \int_{-1}^1 \int_1^3 x^2 y + \cos z \; dz\,dy\,dx$

Problem Describing regions.

Ex Type 1: $\{(x,y,z) \mid (x,y) \in D \quad u_1(x,y) \le z \le u_2(x,y)\}$



Rmk D can be type I or II!

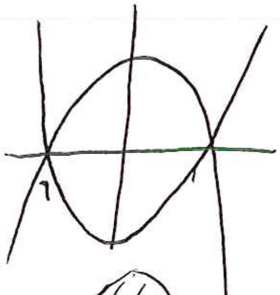
Type 2 $\{(x,y,z) \mid (y,z) \in D, \quad u_1(y,z) \le x \le u_2(y,z)\}$

Type 3 ...

6 possible regions

Ex E is region above region bounded by $y = 1 - x^2$, $y = x^2 - 1$ and below $z = 16 - x^2 - y^2$.

Find $\iiint_E xy\,dV$

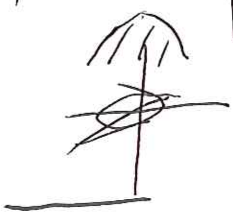


D: $-1 \le x \le 1$

$x^2-1 \le y \le 1-x^2$

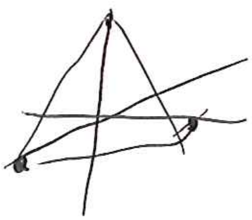
$0 \le z \le 16-x^2-y^2$

$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2}\int_{0}^{16-x^2-y^2} xy\,dz\,dy\,dx$$

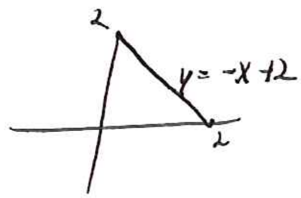


Ex $\displaystyle\int_0^1\int_y^{2y}\int_0^{x+y} 6xz\,dz\,dx\,dy$

Ex $\displaystyle\iiint_T y^2\,dV$ T tetrahedron w/ vertices (0,0,0) (2,0,0) (0,2,0) (0,0,



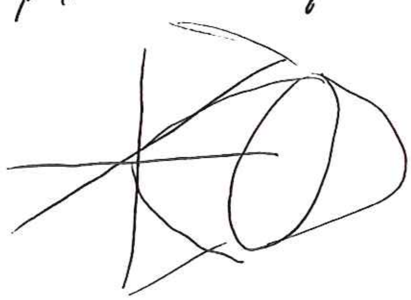
Floor



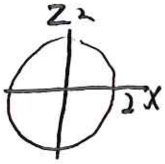
$$\int_0^2\int_0^{2-x}\int_0^{2-x-y} y^2\,dz\,dy\,dx$$

ceiling $x+y+z=2$

Ex Find volume enclosed by paraboloids $y=x^2+z^2$ & $y=8-x^2-z^2$.



∩ when $8-x^2-z^2=x^2+z^2 \Rightarrow x^2+z^2=4$, $y=4$

D= 

$$\iint_D \int_{x^2+z^2}^{8-x^2-z^2} dy\,dA$$ use polar = cylindrical coord

$$\int_0^{2\pi}\int_0^{2}\int_{r^2}^{8-r^2} r\,dz\,dr\,d\theta = \ldots$$

EX $\int_0^1 \int_y^1 \int_0^y f(x,y,z)\,dz\,dx\,dy$

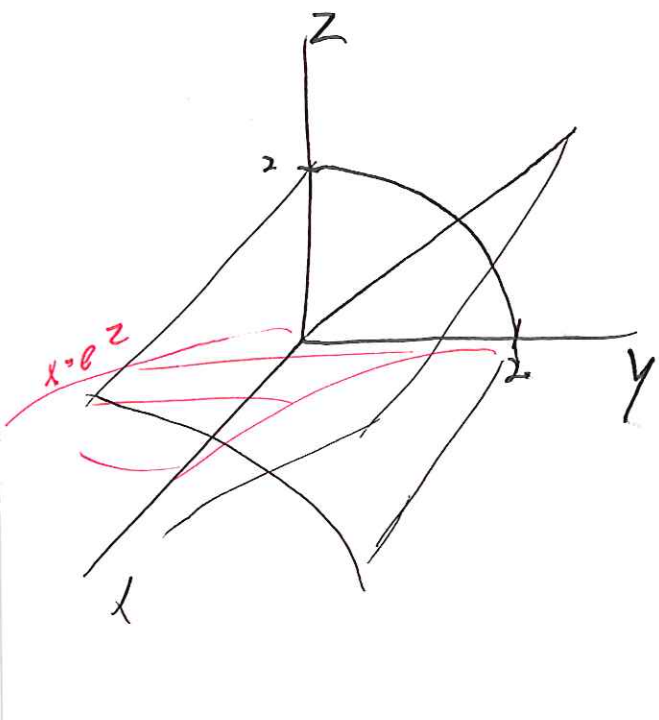
Write 5 other iterated integrals w/ same answer.

EX

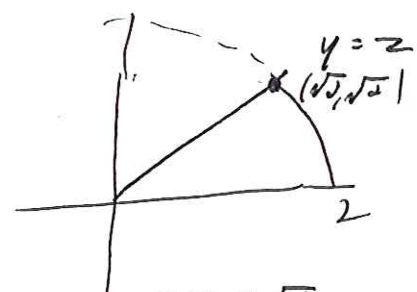
Evaluate $\iiint_E 1/x \, dV$

E is 1^{st} quadrant region bounded by plane $x=1$, below by cylinder $x=e^z$

above by plane $y=z$ and cylinder $y^2+z^2=4$



$1 \le x \le e^z$



$0 \le z \le \sqrt{2}$

$z \le y \le \sqrt{4-z^2}$