Lecture 15 Review: Calculating double integrals using iterated integrals.

Ex $\int_1^4 \int_0^2 6x^2y - 2x \, dy\, dx$ do both ways

Ex $\iint_R ye^{-xy}\, dA$ $R = [0,2] \times [0,2] = \int_0^2 \int_0^2 ye^{-xy}\, dx\, dy = \int_0^2 -e^{-xy}\Big|_{x=0}^{2} dy$

$= \int_0^2 (-e^{-2y} + 1)\, dy$

$= \frac{1}{2}e^{-2y} + y\Big|_0^2 =$

Ex Find volume of solid under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above $R = [-1,1] \times [1,2]$.

Fubini Thm Suppose $f(x,y)$ is continuous on R: $a \le x \le b$ $c \le y \le d$ Then

$$\iint f(x) \, dA = \int_a^b \int_c^d f(x,y)\, dy\, dx = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

Ex $\iint_R y \sin(xy)\, dA$ $R = [1,2] \times [0,\pi]$

* choose direction that makes easier integral.

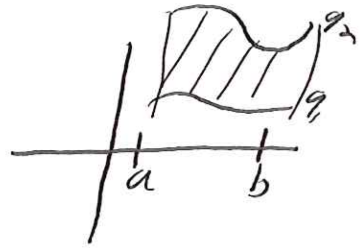
Average Value Recall $y = f(x)$, $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\, dx$.

Def The average value of $f(x,y)$ on R is $\frac{1}{\text{Area } R} \iint_R f(x,y)\, dA$

Ex $\iint \frac{xy}{1+x^4}\, dA$ $R = \{(x,y) \mid -1 \le x \le 1 \quad 0 \le y \le 1\}$

Why is this zero?

Double integrals over general regions



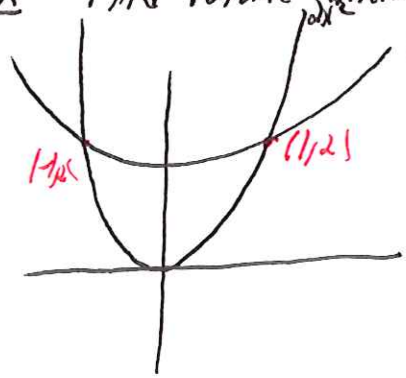
Idea 1. Describe region as $a \le x \le b \quad g_1(x) \le y \le g_2(x)$

or

$c \le y \le b \quad h_1(y) \le x \le h_2(y)$

2. Set up iterated integral in proper order

Ex Find volume under $z = x^2 + y^2$ and above D, D bounded by $y = 2x^2$, $y = 1 + x^2$



$-1 \le x \le 1$

$2x^2 \le y \le 1 + x^2$

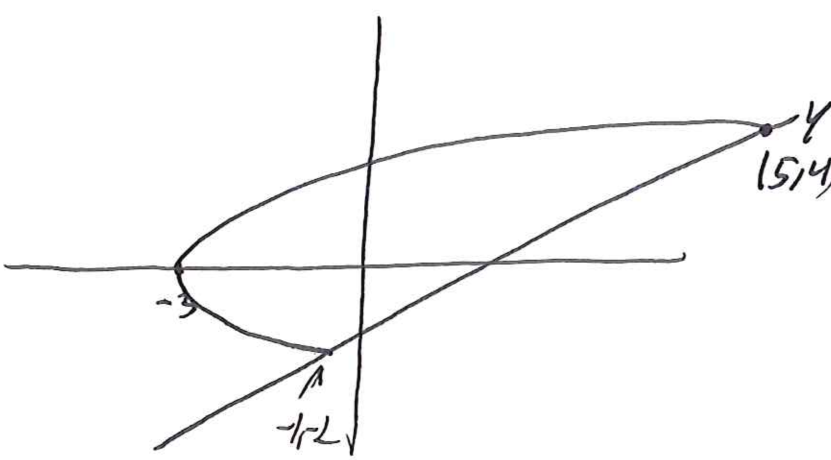
$$\int_{-1}^{1} \int_{2x^2}^{1+x^2} x^2 + y^2 \, dy \, dx$$

$$= \int_{-1}^{1} x^2 y + \frac{y^3}{3} \Big|_{y=2x^2}^{y=1+x^2} dx = \int_{-1}^{1} x^2(1+x^2) + \frac{(1+x^2)^3}{3} - 2x^4 - \frac{8}{3}x^6 \, dx$$

$= ??$

Ex Find volume under $z = xy$ and above triangle $(1,1)$, $(4,1)$, $(1,2)$.

Ex $\iint_D xy \, dA$ D is region bounded by $y = x - 1$, $y^2 = 2x + 6$



which way to set up?

Ex $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$ - swap order to make integral easier.

Ex $\int_1^2 \int_0^{\ln x} f(x,y)\, dy\, dx$ - sketch region & change order.

Ex $f(x,y) = x \sin y$, D enclosed by $y=0$, $y=x^2$, $x=1$

Find avg value

Basic Properties

1. $\iint_D f(x,y) + g(x,y)\, dA = \ldots$

2. $\iint_D c \cdot f(x,y)\, dx$

3. Suppose $D = D_1 \cup D_2$ where D_1, D_2 overlap only on boundary

Then $\iint_D f(x,y)\, dA = \iint_{D_1} f(x,y)\, dA + \iint_{D_2} f(x,y)\, dA$

Ex 15.2 #58

Def/Thm Area of D is $\iint_D 1\, dA$