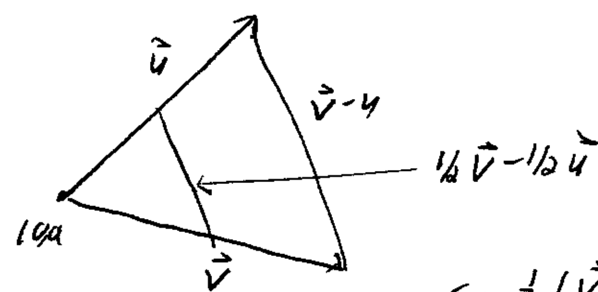
1a. $\cos\theta = \frac{(-1,1,0)\cdot(0,1,-1)}{|(-1,1,0)|\,|(0,1,-1)|} = \frac{1}{\sqrt{2}\sqrt{2}} = 1/2$

$$\boxed{\theta = \cos^{-1}(1/2) = \pi/3}$$

b. $(2,3,-4)\cdot\frac{(1,1,\sqrt{2})}{|(1,1,\sqrt{2})|} = \frac{5-4\sqrt{2}}{\sqrt{1+1+2}} = \frac{5}{2} - 2\sqrt{2}$

2. a. $\vec{v}-\vec{u}$

b.



so $\frac{1}{2}(\vec{v}-\vec{u})$ is // to $\vec{v}-\vec{u}$ w/ 1/2 the length?

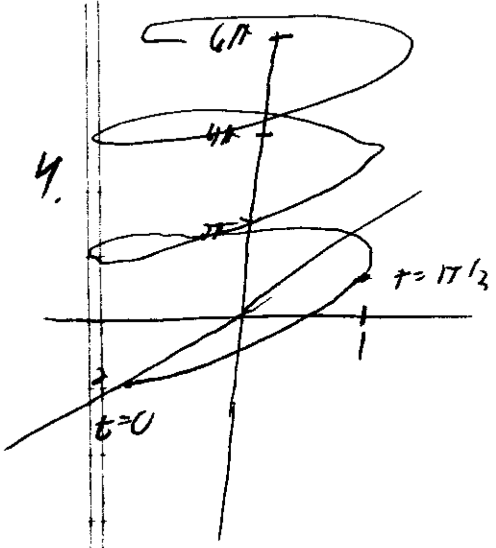
3. $\vec{n} = (1,1,1) - (-2,3,1) = (3,-2,0)$

$$\boxed{3x-2y=10}$$

4. $\vec{v}(t) = (-2\sin t, \cos t, 1)$
$a(t) = (-2\cos t, -\sin t, 0)$
speed $= |v(t)| = \sqrt{4\sin^2 t + \cos^2 t + 1} = \sqrt{3\sin^2 t + 1}$

Sketch ove

4.



3 times around above ellipse

$$\left(\frac{x}{2}\right)^2+y^2=1$$

5. $\operatorname{div}\vec{F} = yz + x\cos y + y$

$\operatorname{curl}\vec{F} = (z,\ xy,\ \sin y - xz)$

6. a. $\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$

$= 2x\cos t + 2y\sin t$

$t=\pi/3 \quad s=2$

$x = 2\cos\pi/3 = 1$

$y = 2\sin\pi/3 = \sqrt{3}$

$= 2 + 2\sqrt{3}\cdot\frac{\sqrt{3}}{2} = 5$

b. $dz = (2x\cos(xy) - x^2y\sin(xy))\,dx - x^3\sin(xy)\,dy$

7. $r'(t) = (1,-1)$

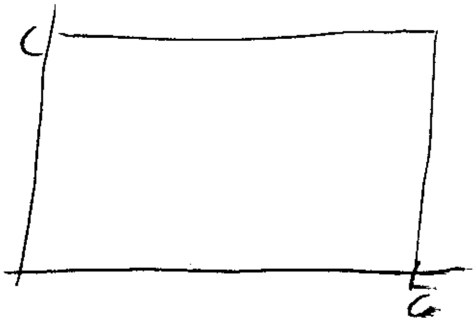
$\int_0^1 (t^2(1-t),\ t^2-(1-t))\cdot(1,-1)\,dt$

$\int_0^1 t^2-t^3-t^2+(1-t) = \int_0^1 1-t-t^3\,dt$

$= t - t^2/2 - t^4/4\Big|_0^1$

$= 1-1/2-1/4 = 1/4$

8. We prove the min of $f(x,y)$ is zero on any square $[0,c]\times[0,c]$, and hence on $[0,\infty)\times[0,\infty)$



$$\nabla f = \left(1-\frac{\cancel{y}}{\sqrt{xy}},\ 1-\frac{x}{\sqrt{xy}}\right)$$

Set $\nabla f = 0$

so only C.P. are $x=y$

C.P. $x=y \to f=0$

boundary: $x=0 \to f=y\geq 0$

$y=0 \to f=x\geq 0$

$x=c \to f=c+y-2\sqrt{cy}$

$$f' = 1-2\cdot\frac{c}{2\sqrt{cy}} = 1-\frac{c}{\sqrt{cy}}$$

for $0\leq c\leq y$ NOTE

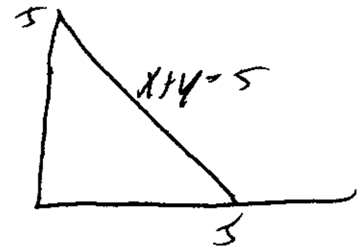
this is increasing $f(c,c)=0$

So global min is 0 on line $x=y$

Thus $x+y-2\sqrt{xy}\geq 0$

$$\Rightarrow \sqrt{xy}\leq\frac{x+y}{2}$$

9.



$f = 3 + xy - x - 2y$ $\quad \nabla f = (y-1,\ x-2)$

C.P. $(2,1)$

Boundary: $x=0,\ 0 \le y \le 5$ $\quad f = 3-2y \quad f' = -2$ never 0

test $(0,0)$ $(0,5)$

$y=0 \quad 0 \le x \le 5$ $\quad f = 3-x \quad f' = -1$ never 0

test $(0,0)$ $(5,0)$

$x = 5-y \quad 0 \le y \le 5$

$f = 3 + (5-y)y - (5-y) - 2y$

$= 3 - y^2 + 5y - 5 + y - 2y$

$= -2 - y^2 + 4y \quad f' = -2y + 4$

$0 = -2y + 4 \quad y = 2$

C.P. $(3,2)$

pt	$f(x,y)$	
$(0,0)$	3	← Global max
$(0,5)$	−7	← Global min
$(5,0)$	−2	
$(2,1)$	1	
$(3,2)$	2	

10. Use spherical coor: $2 \le \rho \le 3$ $0 \le \phi \le \pi/2$, $-\pi/2 \le \theta \le \pi/2$

$$\int_2^3 \int_0^{\pi/2} \int_{-\pi/2}^{\pi/2} \rho\cos\theta \sin\phi \; \rho\cos\phi \; \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho$$

$$= \int_2^3 \rho^4 d\rho \cdot \int_0^{\pi/2} \sin^2\phi \cos\phi \, d\phi \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \left(\frac{3^5 - 2^5}{2}\right)\left(\frac{1}{3}\sin^3\phi\right)\Big|_0^{\pi/2} \sin\theta \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{211}{2} \cdot \frac{1}{3} \cdot (2) = 211/3$$

11. a. curl $\vec{F} = 0$ ✓

b. $f = xy + xz + g(y,z)$
$f = xy + y^2 + g(x,z)$
$f = -\cos z + xz + g(x,y)$

$\boxed{f = xy + xz + y^2 - \cos z}$

c. $W = f(r(3)) - f(r(0))$
$= f(3e^3, \frac{2}{8}, 18\pi) - f(0, 1/5, 0)$

$= \left(\frac{3}{4}e^3 + 54\pi e^3 + \frac{1}{64} - 1\right) - \left(1/25 - 1\right)$

by FT on line S

12. $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 3x^2 + 3y^2$

curve

G.T. says Answer 13

since clockwise

$$= -\iint_D 3x^2 + 3y^2$$

$$= -\int_0^1 \int_0^{2\pi} 3r^2 r \, dr \, d\theta$$

$$= -\int_0^1 6\pi r^3 \, dr$$

$$= -\frac{6\pi r^4}{4}\Big|_0^1$$

$$= -3\pi/2$$

13. $r_u = (\cos v, \sin v, 0)$

$r_v = (-u\sin v, u\cos v, 1)$ $\quad r_u \times r_v = (\sin v, -\cos v, u)$

$$|r_u \times r_v| = \sqrt{\sin^2 v + \cos^2 v + u} = \sqrt{u+1}$$

$$S.A. = \int_0^1 \int_0^{\pi} \sqrt{u+1} \, dv \, du = \int_0^1 \pi\sqrt{u+1} \, du$$

$$= \frac{2}{3}\pi(u+1)^{3/2}\Big|_0^1$$

$$= \frac{2}{3}\pi\left(2^{3/2} - 1\right)$$

14. ~~Area~~ $= \frac{1}{2}$

14. Greens Thm can be used here

$$\text{Area} = \frac{1}{2}\left(\oint x\,dy - y\,dx\right)$$

$$\vec{r}(t) = \left(\frac{\cos t}{2}, \sin t\right) \quad 0 \le t \le 2\pi$$

$$\frac{1}{2}\int_0^{2\pi} \frac{1}{2}\cos t \cdot \cos t\,dt - \sin t \cdot \left(-\frac{1}{2}\sin t\right)dt$$

$$= \frac{1}{2}\int_0^{2\pi} \frac{1}{2}(\cos^2 + \sin^2) = \frac{1}{2}\int_0^{2\pi} \frac{1}{2}\,d\theta = \cancel{\pi/3}\ \pi/2$$