Name:

Math 241WW- Final Exam - December 12, 2008

Instructions: The exam is worth 150 points. Calculators are not permitted. You are permitted two index cards, each no larger than 3×5, with whatever you like written on them.

1. **(10 points)** Let $f(x, y) = x^2 + 2y^2$. Sketch level curves $f(x, y) = k$ for $k = 1, 2, 3, 4$ and on the same picture sketch the vector field ∇f.

b. Find the direction and the rate of maximum increase for $f(x, y)$ at the point $(1, 3)$.

2. **(15 points)** Let S be the surface $z = x^2 + y^2$ lying below the plane $z = 2$.

a. Neatly sketch the surface.
b. Parameterize the surface as $\vec{r}(u, v)$ with $(u, v) \in D$. Be sure to describe the region D.
c. Let $\vec{F}(x, y, z) = (z, x, y)$. Use Stokes' theorem to evaluate:

$$\iint_S \operatorname{curl} \vec{F} \cdot \vec{n} \, dS.$$

3. **(15 points)** Let C be the curve of intersection of the plane $y + z = 7$ and the cylinder $x^2 + y^2 = 4$, oriented to be counterclockwise when viewed from above. Use Stokes' theorem to compute $\int_C \vec{F} \cdot d\vec{r}$ where $F(x, y, z) = (-y^2, x, z^2)$.

4. **(15 points)** Let S be the cylinder (including top and bottom) $x^2 + y^2 = 1, 0 \leq z \leq 1$, with outward pointing normal. Let $\vec{F}(x, y, z) = (x^3, y^3, e^{-z})$. Use the divergence theorem to compute

$$\int\int_S \vec{F} \cdot \vec{n}\, dS.$$

5. **(10 points)** Let S be the portion of the paraboloid $z = x^2 + y^2$ which lies between the planes $z = 1$ and $z = 4$. Write down *but do not evaluate* an integral which gives the surface area of S.

6. **(10 points)** Consider the surface parameterized by $\vec{r}(u,v) = (u^2 + v^2, uv, uv^2)$. Find the tangent plane to this surface at the point where $u = 1, v = 2$. Give the equation of the tangent plane in two forms, first parameterize it. Second, give it in $ax + by + cz = d$ form.

7. **(10 points)** Use Green's theorem to calculate the work done by the force field $\vec{F}(x, y) = (x^2 - y^3, x^3 + y^2)$ moving a particle around the unit circle $x^2 + y^2 = 1$ in the *clockwise direction.*

8. **(10 points)** Let $f(x, y, z) = x^2y + y^2z + xyz$.

a. Calculate ∇f and $\text{curl}(\nabla f)$.

b. Calculate the line integral: $\int_C \nabla f \cdot d\vec{r}$ where C is the top half of the ellipse $3x^2 + 7y^2 = 12$ traversed from $(-2, 0)$ to $(2, 0)$.

9. **(10 points)** Let $f(x, y) = x^4 + y^2 - 8x^2 - 6y + 16$. Find and classify all critical points of $f(x, y)$ as local max, local min or saddle points.

10. **(10 points)** Suppose $w = x^2 e^{yz}$ and $(x, y, z) = (st + u^2, s^2ut, s + tu)$. Find $\frac{\partial w}{\partial u}$ at the point where $s = 1, t = 2, u = 3$.

11. **(10 points)** Find the equation of the plane containing the points $(1, 2, 3), (-2, 1, 1), (-3, 1, 2)$.

12. **(10 points)** Let D be the region enclosed by the curves $y = 0$, $y = x^2$ and $x = 1$. Find the average value of $f(x, y) = x \sin y$ over the region D.

13. **(5 points)** Consider a particle with position at time t given by $\vec{r}(t) = (1+t, t^3, t^2)$. What is the speed of the particle at when $t = 3$?

14. **(10 points)** Let R be the rectangle $[-3, 3] \times [0, 3]$. Estimate $\int\int_R xy^2$ using a Riemann sum with $m = n = 3$ and the upper right corner of each rectangle as your sample point.