

Name:

Math 241- Midterm Exam #3 - November 13, 2008

**Instructions:** You are allowed a single 3" by 5" index card but no other notes, books, or calculators.

1. (15 points) Use spherical coordinates to evaluate:

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} dz dx dy$$

2. (10 points) Draw a sketch of the region  $D$  over which the iterated integral is being evaluated, then express it as an iterated integral in which the order of integration is reversed:

$$\int_0^2 \int_0^{x^2} f(x, y) dy dx.$$

3. **(15 points)** Find the work done by a force  $\vec{F}(x, y) = (2 - y, x)$  in moving a particle along one arch of the cycloid given by  $\vec{r}(t) = (t - \sin t, 1 - \cos t)$  for  $0 \leq t \leq 2\pi$ .

4. **(5 points)** A vector field  $\vec{F}$  is *conservative* if ...

5. **(10 points)** Neatly sketch the vector field below, be sure to sketch enough vectors so the behavior of the vector field is clear.

$$\vec{F}(x, y) = (-y, x)$$

6. **(10 points)** Find the volume of the part of the ball  $\rho \leq 5$  that lies between the cones  $\phi = \pi/4$  and  $\phi = \pi/3$ .

7. **(10 points)** Let  $R$  be the rectangle  $[0, 4] \times [0, 2]$ . Estimate  $\int \int_R x^2 y$  using a Riemann sum with  $m = n = 2$  and the upper right corner of each rectangle as your sample point.

8. (15 points) Find the volume of the region  $E \subset \mathbb{R}^3$  bounded below by the  $xy$ -plane, above by the plane  $z = x$ , and by the parabolic cylinder  $y^2 = 4 - x$ .

9. (10 points) Evaluate the double integral:

$$\iint_D xy \, dA$$

where  $D$  is the triangular region with vertices  $(0, 0)$ ,  $(1, 2)$  and  $(0, 3)$ .