Name:

Math 241- Midterm Exam #2 - October 21, 2008

1. **(10 points)** Find the absolute maximum and minimum values of

$$f(x, y) = x^4 + y^4 - 4xy + 9$$

on the set $D = \{(x, y) \mid 0 \le x \le 3, 0 \le y \le 2\}$.

2. (**10 points**) Let $f(x, y) = x^3 - 3x + y^4 - 2y^2$. Find all the critical points of $f(x, y)$ and classify each as a local maximum, local minimum or saddle point.

3. (**10 points**) Find the maximum rate of change of $f(x, y) = xy^2 + \sqrt{x}$ at the point $(1, 3)$. In what direction does it occur?

4. (**10 points**) Consider the surface given by

$$xy + xz + y^2 z + 7x = 19.$$

Find the equation of the tangent plane and the normal line to this surface at the point $(1, 2, 2)$.

5. **(10 points)** Let

$$z^2 + \cos(x) + \frac{y}{z} = 5.$$

Find $\frac{\partial z}{\partial y}$.

6. **(10 points)** Let $f(x, y) = x\sin(xy) + y^3$. Find the directional derivative of $f(x, y)$ at the point $(\pi/2, 1)$ in the direction $(1, 4)$.

7. **(10 points)** Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{xy}.$$

If the limit does not exist, write DNE but be sure to justify your answer.

8. **(10 points)** Let $f(x,y) = \frac{\sqrt{y-x^2}}{x-3}$. Neatly sketch the domain of $f(x,y)$.

9. (**10 points**) Let $f(x, y) = 2x^2 + y^2$. Sketch the level curves $f(x, y) = k$ for $k = 1, 2, 3$.

10. (**10 points**) Suppose $F(x, y) = x^2 y + y^2$, $x = st + v^2 + uv$, $y = s - u^2 v$.

    a. Find $\frac{\partial F}{\partial u}$.

    b. Find $\frac{\partial F}{\partial s}$ when $s = 1, t = 2, u = 3, v = 4$.