Name:

Math 241- Midterm Exam #1 - September 18, 2008

1. **(14 points (2 pts each, no partial credit))** Let $\vec{a} = (-4, 1, 2)$ and $\vec{b} = (1, 2, 3)$.

   a. Find $\vec{a} \times \vec{b}$.

   b. Find $\vec{a} \cdot \vec{b}$.

   c. Determine the magnitudes $|\vec{a}|$ and $|\vec{b}|$.

   d. Let Θ be the angle between $\vec{a}$ and $\vec{b}$. Find $\cos\Theta$.

   e. Find $3\vec{a} - 2\vec{b}$.

f. Find the vector projection $\text{prog}_{\vec{a}}\vec{b}$ of $\vec{b}$ onto $\vec{a}$.

g. Find the area of the triangle with corners $(0,0,0)$, $(-4,1,2)$, $(1,2,3)$.

2. **(5 points)** Given points $A=(1,1,1)$, $B=(2,3,0)$, $C=(-1,1,4)$ and $D=(0,3,2)$, find the volume of the parallelepiped with adjacent edges AB, AC and AD.

3. **(10 points)** Find the equation of the plane containing the points $(2, 1, 1)$, $(3, 0, 2)$ and $(-1, 1, 1)$. Then find the parametric equation of the line passing through $(3, 0, 2)$ and perpendicular to the plane.

4. **(5 points)** Sketch the curve $\vec{r}(t) = (\cos(t), 2\sin(t))$ for $0 \leq t \leq 2\pi$ in the xy-plane. Be sure to label intercepts and indicate with an arrow the direction of increasing t.

5. **(10 points)** The position function of a particle is given by $\vec{r}(t) = (t^2, 5t, t^2 - 16t)$ At what time is its speed a minimum? Hint: Minimizing the square of the speed is easier and clearly gives the same answer.

6. **(10 points)** A particle has acceleration $\vec{a}(t) = (1, -1, t)$, initial velocity $\vec{v}(0) = (1, 1, -1)$ and initial position $\vec{r}(0) = (1, -1, 1)$. Find the equation for its position $\vec{r}(t)$.

7. **(15 points)** Consider the space curve

$$\vec{r}(t) = (\cos(t), \sin(t), t^2).$$

a. Find the unit tangent vector and unit normal vector to the curve at the point $(0, 1, \frac{\pi^2}{4})$.
b. Determine the curvature $\kappa(t)$.
c. Set up but do not evaluate an integral which gives the length of the curve for $1 \leq t \leq 3$.

8. **(5 points)** Find the parametric equation for the tangent line to the curve $x = 1 + 2\sqrt{t}$, $y = t^3 - t$, $z = t^3 + t$ at the point $(3, 0, 2)$.

9. **(10 points)** At what point to the curves $\vec{r}_1(t) = (t, 1-t, 3+t^2)$ and $\vec{r}_2(t) = (3-t, t-2, t^2)$ intersect? What is the cosine of the angle between them at the point of intersection?

10. **(6 points)** Describe a method for determining whether four points P, Q, R and S lie in the same plane.

11. **(10 points)**

a. Neatly sketch the graph of $x^2 + y^2 + z^2/4 = 1$, labeling all intercepts.

b. Neatly sketch the graph of $z = y^2$ in $\mathbb{R}^3$.