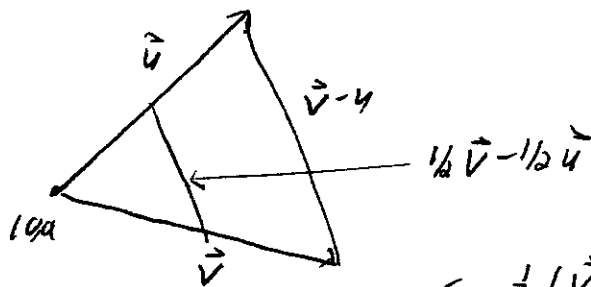


$$1a. \cos \theta = \frac{(-1, 1, 0) \cdot (0, 1, -1)}{|(-1, 1, 0)| |(0, 1, -1)|} = \frac{1}{\sqrt{2}\sqrt{2}} = 1/2$$

$$\theta = \cos^{-1}(1/2) = \pi/3$$

$$b. (2, 3, -4) \cdot \frac{(1, 1, \sqrt{2})}{|(1, 1, \sqrt{2})|} = \frac{5 - 4\sqrt{2}}{\sqrt{1+1+2}} = \frac{5}{2} - 2\sqrt{2}$$

$$2. a. \vec{v} - \vec{u}$$



So $\frac{1}{2}|\vec{v}-\vec{u}|$ is // to $\vec{v}-\vec{u}$ w/ $1/2$ the length

$$3. \vec{n} = (1, 1, 1) - (2, 3, 1) = (3, -2, 0)$$

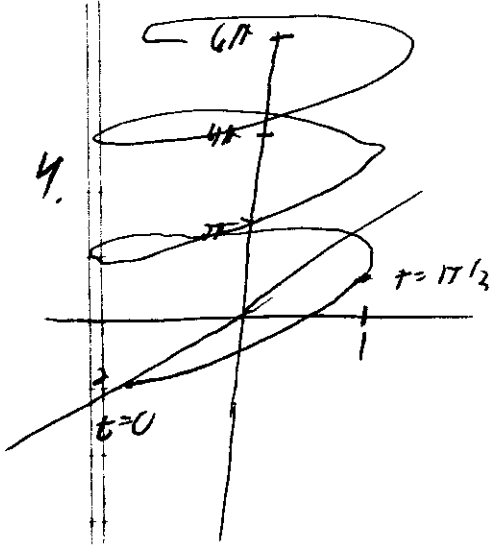
$$3x - 2y = 10$$

$$4. |\vec{v}(t)| = (-2\sin t, \cos t, 1)$$

$$a(t) = (-2\cos t, -\sin t, 0)$$

$$\text{speed} = |\vec{v}(t)| = \sqrt{4\sin^2 t + \cos^2 t + 1} = \sqrt{3\sin^2 t + 1}$$

Sketch one



3 times around above ellipse

$$\left(\frac{x}{2}\right)^2 + y^2 = 1$$

5. $\text{div } \vec{F} = yz + x \cos y + y$

$$\text{curl } \vec{F} = (z, xy, \sin y - xz)$$

6. a. $\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s}$

$$= 2x \cos t + 2y \sin t$$

$$t = \pi/3 \quad s = 2$$

$$x = 2 \cos \pi/3 = 1$$

$$y = 2 \sin \pi/3 = \sqrt{3}$$

$$= 2 + 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = \textcircled{5}$$

b. $dz = (2x \cos(xy) - x^2 y \sin(xy)) dx - x^3 \sin(xy) dy$

$$r'(t) = (1, -1)$$

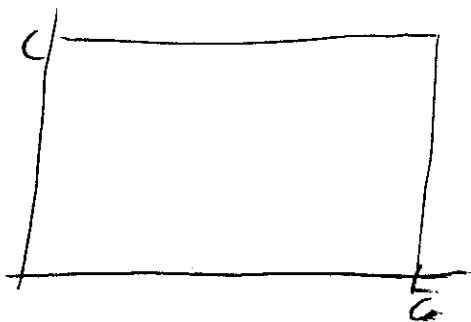
7. $\int_0^1 (t^2(1-t), t^2 - (1-t)) \cdot (1, -1) dt$

$$\int_0^1 (t^2 - t^3 - t^2 + (1-t)) dt = \int_0^1 (1 - t - t^3) dt$$

$$= t - \frac{t^2}{2} - \frac{t^4}{4} \Big|_0^1$$

$$= 1 - \frac{1}{2} - \frac{1}{4} = \textcircled{\frac{1}{4}}$$

8. We prove the min of $f(x,y)$ is zero on any square $[0,c] \times [0,c]$, and hence on $[0,\infty) \times [0,\infty)$



$$\nabla f = \left(1 - \frac{x}{\sqrt{xy}}, 1 - \frac{y}{\sqrt{xy}} \right)$$

Set $\nabla f = 0$

so only C.P. are $x=y$

C.P. $x=y \rightarrow f=0$

boundary: $x=0 \rightarrow f=y \geq 0$

$y=c \rightarrow f=x \geq 0$

$x=c \rightarrow f=c+y-2\sqrt{cy}$

$$f' = 1 - 2 \cdot \frac{c}{2\sqrt{cy}} = 1 - \frac{c}{\sqrt{cy}}$$

for $0 < c \leq y \leq c$ NOT C

THIS IS ~~FALSE~~

INCREASING

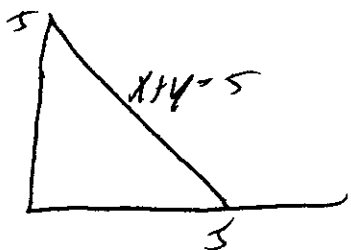
$$f(c,c) = 0$$

so global min is 0 on line $x=y$

Thus $x+y-2\sqrt{xy} \geq 0$

$$\Rightarrow \sqrt{xy} \leq \frac{x+y}{2}$$

9.



$$f = 3 + xy - x - 2y \quad f' = (y-1, x-2)$$

C.P. (2,1)

Boundary : $x=0, 0 \leq y \leq 5$ $f = 3 - 2y$ $f' = -2$ never 0
test (0,0) (0,5)

$y=0, 0 \leq x \leq 5$ $f = 3 - x$ $f' = -1$ never 0
test (0,0) (5,0)

$x=5-y, 0 \leq y \leq 5$

$$f = 3 + (5-y)y - (5-y) - 2y$$

$$= 3 - y^2 + 5y - 5 + y - 2y$$

$$= -2 - y^2 + 4y \quad f' = -2y + 4$$

$$0 = -2y + 4 \quad y = 2$$

C.P. (3,2)

Pt	f(x,y)	
(0,0)	3	Global max
(0,5)	-7	Global min
(5,0)	-2	
(2,1)	1	
(3,2)	2	

10.

Use spherical coord: $2 \leq \rho \leq 3$ $0 \leq \theta \leq \pi/2$, $-\pi/2 \leq \phi \leq \pi/2$

$$\begin{aligned} & \int_2^3 \int_0^{\pi/2} \int_{-\pi/2}^{\pi/2} \rho \cos \theta \sin \phi \rho \cos \phi \rho^2 \sin \theta d\theta d\phi d\rho \\ &= \int_2^3 \rho^4 d\rho \cdot \int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta \cdot \int_{-\pi/2}^{\pi/2} \cos \phi d\phi \\ &= \left(\frac{3^5 - 2^5}{5} \right) \left(\frac{1}{3} \sin^3 \theta \right) \Big|_0^{\pi/2} \sin \theta \Big|_{-\pi/2}^{\pi/2} \\ &= \frac{211}{2} \cdot \frac{1}{3} \cdot (2) = \boxed{211/3} \end{aligned}$$

11. 4. $\text{curl } \vec{F} = 0$ ✓

$$\begin{aligned} b. \quad f &= xy + xz + 9(yz) \\ f &= xy + y^2 + 9(xz) \\ f &= \cos z + xz + 9(xy) \end{aligned}$$

$$f = xy + xz + y^2 - \cos z$$

$$\begin{aligned} c. \quad W &= f(r(3)) - f(r(0)) \\ &= f\left(3e^3, \frac{3}{8}, 18\pi\right) - f(0, 1/5, 0) \\ &= \left(\frac{3}{4}e^3 + 54\pi e^3 + \frac{1}{64} - 1\right) - \left(\frac{1}{25} - 1\right) \end{aligned}$$

by FT on line S

$$12. \quad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 3x^2 + 3y^2$$

correct

since clockwise

G.T. says ANSWER IS

$$- \iint_D 3x^2 + 3y^2$$

$$= - \int_0^1 \int_0^{2\pi} 3r^2 r \, d\theta \, dr$$

$$= - \int_0^1 6\pi r^3 \, dr$$

$$= - \frac{6\pi r^4}{4} \Big|_0^1$$

$$= -3\pi/2$$

$$13. \quad r_u = (\cos v, \sin v, u)$$

$$r_v = (-u \sin v, u \cos v, 1)$$

$$r_u \times r_v = (\sin v, -\cos v, u)$$

$$|r_u \times r_v| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{u^2 + 1}$$

$$S.A. = \int_0^1 \int_0^{2\pi} \sqrt{u^2 + 1} \, dv \, du = \int_0^1 2\pi \sqrt{u^2 + 1} \, du$$

$$= \frac{2}{3} \pi (u^2 + 1)^{3/2} \Big|_0^1$$

$$= \frac{2}{3} \pi (2^{3/2} - 1)$$

$$14. \text{ Area} = \frac{1}{2}$$

14. Green's Theorem can be used here

$$\text{Area} = \frac{1}{2} (\int x dy - y dx)$$

$$\vec{r}(t) = \left(\frac{\cos t}{2}, \sin t \right) \quad 0 \leq t \leq 2\pi$$

$$\frac{1}{2} \int_0^{2\pi} \frac{1}{2} \cos t \cdot \cos t dt - \sin t \cdot \left(-\frac{1}{2} \sin t \right) dt$$

$$= \frac{1}{2} \int_0^{2\pi} \frac{1}{2} (\cos^2 + \sin^2) = \frac{1}{2} \int_0^{2\pi} \frac{1}{2} dt = \pi/2$$