Name: SOLUTIONS

Math 2950- Midterm Exam #2 - October 29, 2004

1. **(15 points)** Find $\partial z/\partial s$ at the point where $s = 1$ and $t = 2$.

$$z = 2xy + x^2, \;\; x = s + t^2, \;\; y = st$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (2y+2x)(1) + 2x\,t$$

$s=1$ $t=2$ $x=5$ $y=2$

$$= 14 \cdot 1 + 20 = 34$$

2. **(15 points)** Let $f(x, y) = 4x^2 - xy + y^2$. Find the critical point(s) of $f(x, y)$. Use the second derivative test to classify each as a local max, local min or saddle.

$\nabla f = (8x - y, -x + 2y)$

set $\nabla f = (0,0)$ gives $x=0, y=0$ as only C.P.

$$D = \begin{vmatrix} 8 & -1 \\ -1 & 2 \end{vmatrix} = 15 > 0$$

$f_{xx} = 8 > 0$

local min

3. **(10 points)** Your grade g on this exam depends on the average amount of time t that you study each day and your attendance a since the last exam. Values of the function $g = f(a,t)$ are given in the following table where t is given in hours and a is given in days.

a\t	1	2	3	4	5
20	50	55	61	68	76
21	55	61	68	76	84
22	60	64	72	77	87
23	63	67	76	81	89
24	66	70	82	86	95

Estimate the values of $f_a(22,3)$ and $f_t(23,4)$.

15   55

4   6.5

4. **(15 points)**. Write the equation of the tangent plane to the surface $x^2 - 2yz - z^2 = 1$ at the point $(1,-1,2)$.

$\nabla f = (2x, -2z, -2y-2z)$

at $(1,-1,2) = (2,-4,-2)$

$(2,-4,-2)\cdot(x-1, y+1, z-2) = 0$

5. **(10 points)** Find the total differential dz:

$$z = y\cos(xy)$$

$dz = -y^2\cos(xy)\,dx + (\cos(xy) - xy\sin(xy))\,dy$

6. **(15 points)** Find the extreme values of the function $f(x,y) = e^{-xy}$ on the region $x^2 + 4y^2 \leq 1$.

$\nabla f = (-ye^{-xy}, -xe^{-xy})$ This is (0,0) only at (0,0)

so (0,0) is only C.P. Now use Lagrange mult. on boundary

$\nabla g = (2x, 8y)$

$-ye^{-xy} = 2x\lambda \quad -xe^{-xy} = 8y\lambda \quad x^2+4y^2=1$

$\lambda = \frac{-y}{2x}e^{-xy} \quad \lambda = \frac{-x}{8y}e^{-xy}$

$\frac{-y}{2x} = \frac{-x}{8y} \quad 8y^2 = 2x^2$

$x = \pm 2y$

$x = 2y$ gives $8y^2 = 1 \quad y = \pm 1/\sqrt{8}$

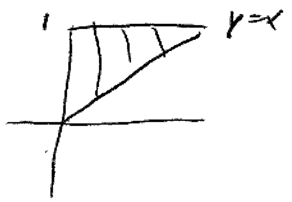
$x = -2y$ " " "

So $(\pm 2/\sqrt{8}, \pm 1/\sqrt{8})$

pt	f
(0,0)	1
$(2/\sqrt{8}, 1/\sqrt{8})$	$e^{-1/4}$ ← min
$(2/\sqrt{8}, -1/\sqrt{8})$	$e^{1/4}$ ← max
$(-2/\sqrt{8}, 1/\sqrt{8})$	$e^{1/4}$ ← max
$(-2/\sqrt{8}, -1/\sqrt{8})$	$e^{-1/4}$ ← min

7. **(10 points)** Evaluate the double integral by first reversing the order of integration

$$\int_0^1 \int_x^1 \cos(y^2)\,dy\,dx.$$



$$\int_0^1 \int_0^y \cos(y^2)\,dx\,dy$$

$$= \int_0^1 y\cos(y^2)\,dy$$

$$= \frac{1}{2}\sin(y^2)\Big|_0^1$$

$$= \frac{1}{2}\sin 1$$

8. **(10 points)** Compute $\int_D \int f(x,y)dA$ where $f(x,y) = 1/\sqrt{x^2+y^2}$ and D is the first quadrant region lying between the circles $x^2+y^2=4$ and $x^2+y^2=9$.

In polar we get $0 \le \theta \le 2\pi$ $\quad 2 \le r \le 3$

$$\int_0^{2\pi} \int_2^3 \frac{1}{r}\, r\,dr\,d\theta = \int_0^{2\pi} \int_2^3 dr\,d\theta$$

$$= 2\pi$$