**Name:**

**Math 2950- Midterm Exam #1 - September 27, 2004**

1. **(10 points)** Find the equation of the plane perpendicular to the line

$$(x, y, z) = (1, -2, 3) + t(1, -2, 1)$$

and passing through the point $(2, 4, -1)$.

2. **(15 points)** Let $\vec{u} = (1, -1, 3)$, $\vec{v} = (1, 1, 2)$. Calculate $\vec{u} \cdot \vec{v}$ and $\vec{u} \times \vec{v}$. Then find the angle between $\vec{u}$ and $\vec{v}$.

3. (**10 points**). Let $\vec{u}$ be a differentiable vector funtion and $f$ a real valued function. Then the chain rule says:

$$\frac{d}{dt}[\vec{u}(f(t))] = ?$$

4. **(10 points)** Sketch the space curve

$$\vec{r}(t) = (t, 2t, \cos(t))$$

for the interval $0 \leq t \leq 3\pi$, indicating with an arrow the direction of increasing t.

5. **(10 points)** Sketch the region given by the spherical coordinate inequalities:

$$\begin{aligned}
&0 \leq \phi \leq \pi/2 \\
&-\pi/2 \leq \theta \leq \pi/2 \\
&0 \leq \rho \leq 2.
\end{aligned}$$

6. **(20 points)**

a. Sketch the surface given by $z = x^2 + y^2$.

b. Verify that the two space curves below both lie on the surface:

$$r(\vec{t}) = (\cos(t), \sin(t), 1) \text{ and } \vec{x}(s) = (s, 0, s^2)$$

c. Verify that the point $(1, 0, 1)$ lies on both curves. (i.e. find a $t_0$ such that $(1, 0, 1) = \vec{r}(t_0)$ and a $s_0$ such that $(1, 0, 1) = \vec{x}(s_0)$.)

d. Find the equation for the tangent line to the curve $\vec{r}(t)$ at the point $(1, 0, 1)$.

e. Find the equation for the tangent line to the curve $\vec{x}(s)$ at the point $(1, 0, 1)$.

f. Find the equation of the plane passing through the point $(1, 0, 1)$ and containing both tangent lines above. This is called the *tangent plane* to the surface at the point $(1, 0, 1)$.

7. **(15 points)** a. Find the velocity, acceleration, and speed of a particle with position function given by:

$$r(t) = (t^2 + 1, \cos(t), t)$$

b. Express the arc length of the curve above from $t = 1$ to $t = 2$ as a definite integral (do not try to evaluate the integral!).

8. (**10 points**) Find the work by a force $\vec{F}= (1, -1, 1)$ moving an object from the point $(1, 1, 1)$ to the point $(4, 2, 1)$.