

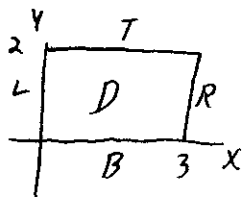
Name: SOLUTIONS

Math 241- Midterm Exam #2 - October 21, 2008

1. (10 points) Find the absolute maximum and minimum values of

$$f(x, y) = x^4 + y^4 - 4xy + 9$$

on the set $D = \{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 2\}$.



$$f_x = 4x^3 - 4y \quad f_y = 4y^3 - 4x, \text{ set both} = 0 \text{ gives}$$

$$y = x^3, x = y^3 \text{ so } x = x^9 \text{ so } x = 0, 1 \text{ or } -1. \text{ Thus}$$

Critical points (0,0), (1,1), (-1,-1)

On L, $x=0$ so $f = y^4 + 9$ $f' = 4y^3$, no C.P. test endpoints (0,0), (0,2)

On B, $y=0$ so $f = x^4 + 9$ $f' = 4x^3$, no C.P. test endpoints (0,0), (3,0)

On R, $x=3$ so $f = 81 + y^4 - 12y + 9$, $f' = 4y^3 - 12$

$$4y^3 - 12 = 0$$

$$y^3 = 3$$

$$y = \sqrt[3]{3} \text{ test } (3, \sqrt[3]{3}), (3,0), (3,2)$$

On T, $y=2$ so $f = x^4 + 16 - 8x + 9$, $f' = 4x^3 - 8$ test $(\sqrt[3]{2}, 2), (0,2), (3,2)$

$$4x^3 - 8 = 0$$

$$x = \sqrt[3]{2}$$

point	f value
(0,0)	9
(1,1)	7
(-1,-1)	7
(0,2)	25
(3,2)	82
(3,0)	90
$(3, \sqrt[3]{3})$	$81 + 3\sqrt[3]{3} - 12\sqrt[3]{3} + 9 = 90 - 9\sqrt[3]{3}$
$(\sqrt[3]{2}, 2)$	$2\sqrt[3]{2} + 16 - 8\sqrt[3]{2} + 9 = 25 - 6\sqrt[3]{2}$

Max value = 90

Min value = 7

2. (10 points) Let $f(x, y) = x^3 - 3x + y^4 - 2y^2$. Find all the critical points of $f(x, y)$ and classify each as a local maximum, local minimum or saddle point.

$$f_x = 3x^2 - 3, \quad f_y = 4y^3 - 4y, \quad f_{xx} = 6x, \quad f_{yy} = 12y^2 - 4, \quad f_{xy} = f_{yx} = 0$$

1. Find C.P. Need $f_x = 0, f_y = 0$ gives $x = \pm 1, y = 0, 1, -1$

so $(1, 0), (1, 1), (1, -1), (-1, 0), (-1, 1), (-1, -1)$ are all C.P.

2. $D = \cancel{0} \cdot (12y^2 - 4) - 0 = 6x(12y^2 - 4)$

C.P.	D	f_{xx}	Classify
(1, 0)	-24	N/A	saddle
(1, 1)	48	6	local min
(1, -1)	48	6	local min
(-1, 0)	24	-6	local max
(-1, 1)	-48	N/A	saddle
(-1, -1)	-48	N/A	saddle

3. (10 points) Find the maximum rate of change of $f(x, y) = xy^2 + \sqrt{x}$ at the point $(1, 3)$. In what direction does it occur?

$$\nabla f = (y^2 + \frac{1}{2\sqrt{x}}, 2xy)$$

$$\nabla f(1, 3) = (9 + \frac{1}{2}, 6) = \boxed{(\frac{19}{2}, 6) \leftarrow \text{direction}}$$

$$|\nabla f(1, 3)| = \boxed{\sqrt{(\frac{19}{2})^2 + 36} \leftarrow \text{max rate}}$$

4. (10 points) Consider the surface given by

$$xy + xz + y^2z + 7x = 19.$$

Find the equation of the tangent plane and the normal line to this surface at the point $(1, 2, 2)$.

This is level surface $F(x, y, z) = 19$.

$$F_x = y + z + 7 \quad F_y = x + 2yz \quad F_z = x + y^2$$

Now plug in $(1, 2, 2)$

$$F_x = 11 \quad F_y = 9 \quad F_z = 5 \leftarrow \text{normal vector}$$

$$\text{Tangent plane: } 11(x-1) + 9(y-2) + 5(z-2) = 0$$

$$\text{Normal line: } (1, 2, 2) + t(11, 9, 5)$$

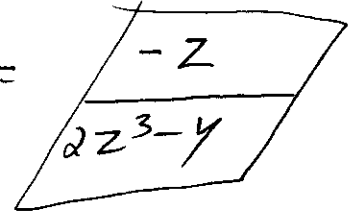
5. (10 points) Let

$$z^2 + \cos(x) + \frac{y}{z} = 5.$$

Find $\frac{\partial z}{\partial y}$.

$$F_y = 1/z \quad F_z = 2z - \frac{y}{z^2}$$

$$\frac{dz}{dy} = - \frac{F_y}{F_z} = \frac{-1/z}{2z - \frac{y}{z^2}} =$$


$$\frac{-z}{2z^3 - y}$$

6. (10 points) Let $f(x, y) = x \sin(xy) + y^3$. Find the directional derivative of $f(x, y)$ at the point $(\pi/2, 1)$ in the direction $(1, 4)$.

$$\vec{u} = \frac{(1, 4)}{\sqrt{17}} = \left(\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right)$$

$$\nabla f = (\sin(xy) + xy \cos(xy), x^2 \cos(xy) + 3y^2)$$

$$\begin{aligned} \nabla f(\pi/2, 1) &= (\sin(\pi/2) + \pi/2 \cos(\pi/2), \pi^2/4 \cos(\pi/2) + 3) \\ &= (1 + 0, 0 + 3) = (1, 3) \end{aligned}$$

$$D_{\vec{u}} = (1, 3) \cdot \left(\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right) = \frac{13}{\sqrt{17}}$$

7. (10 points) Evaluate

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{xy}$$

If the limit does not exist, write DNE but be sure to justify your answer.

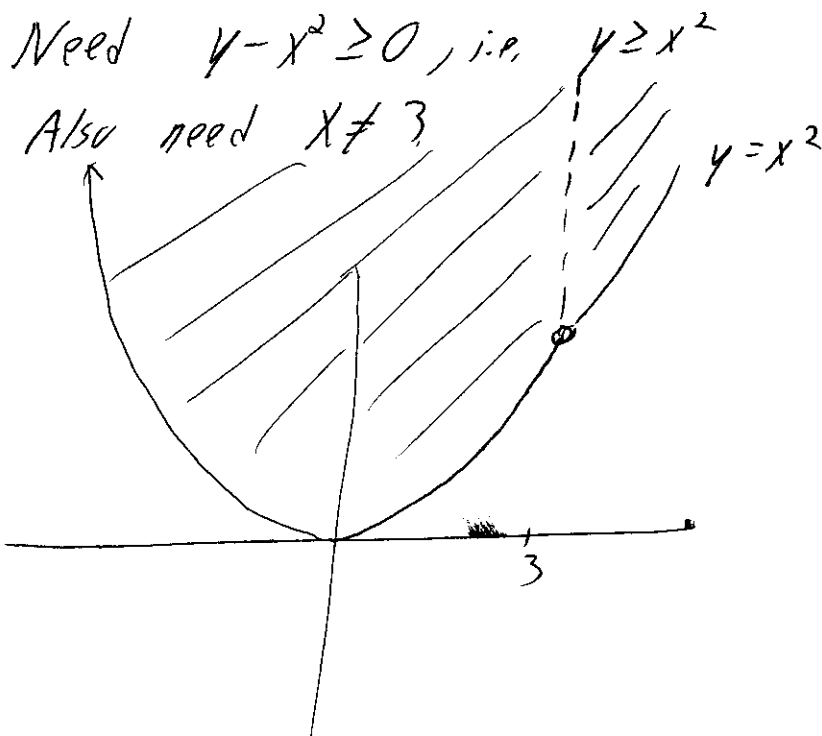
On the line $y=x$, $f(x,y) = \frac{x^2 + x^2}{x^2} = \frac{2x^2}{x^2} = 2$

On the line $y=-x$, $f(x,y) = \frac{x^2 + (-x)^2}{-x^2} = -2$,

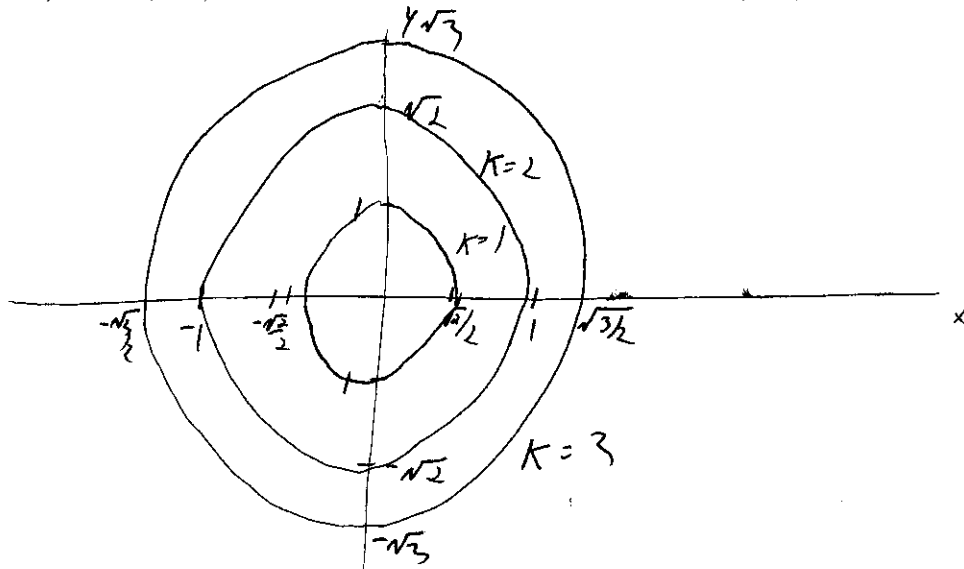
If the limit was $=L$ then the function would approach L along any curve approaching $(0,0)$.

Thus limit **DNE**.

8. (10 points) Let $f(x,y) = \frac{\sqrt{y-x^2}}{x-3}$. Neatly sketch the domain of $f(x,y)$.



9. (10 points) Let $f(x, y) = 2x^2 + y^2$. Sketch the level curves $f(x, y) = k$ for $k = 1, 2, 3$.



10. (10 points) Suppose $F(x, y) = x^2y + y^2$, $x = st + v^2 + uv$, $y = s - u^2v$.

a. Find $\frac{\partial F}{\partial u}$.

b. Find $\frac{\partial F}{\partial s}$ when $s = 1, t = 2, u = 3, v = 4$.

a.
$$\frac{\partial F}{\partial u} = \frac{\partial F}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial F}{\partial y} \frac{\partial y}{\partial u} = (2xy)(v) + (x^2 + 2y)(-2uv)$$

b.
$$\frac{\partial F}{\partial s} = \frac{\partial F}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial F}{\partial y} \frac{\partial y}{\partial s} = (2xy)(t) + (x^2 + 2y)(1)$$

$$x = 18 \quad y = -35$$

$\frac{\partial F}{\partial s}$ at this point is

$$2 \cdot (18)(-35) + (18^2 - 70) = -1006$$