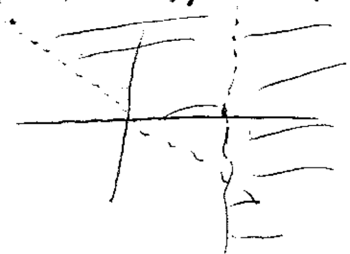
Lecture 8

Parameterized curves $\leftrightarrow$ $\vec{r}(t): \mathbb{R} \to \mathbb{R}^n$ Domain $\subseteq \mathbb{R}$, range $\subseteq \mathbb{R}^n$

Chapter 14 Functions $\mathbb{R}^n \to \mathbb{R}$, i.e. many variables, output is real #

Example

1. $f(x,y) = \dfrac{\ln(x+y)}{x-1}$ Domain: Need $x+y>0$ $x \neq 1$



2. $f(x,y) = \sqrt{y-x^2}$ Domain

3. Find domain & range of $f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$

Domain: $1-x^2-y^2-z^2 \geq 0$

$1 \geq x^2+y^2+z^2$ inside sphere

Range $[0,1]$

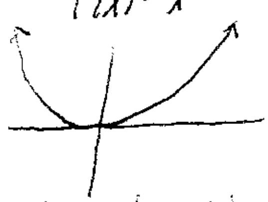
Graphing

Suppose $f: \mathbb{R}^2 \to \mathbb{R}$, we usually plot graph as

$$\text{Graph } f = \{(x,y,z) \in \mathbb{R}^3 \mid z = f(x,y)\}$$

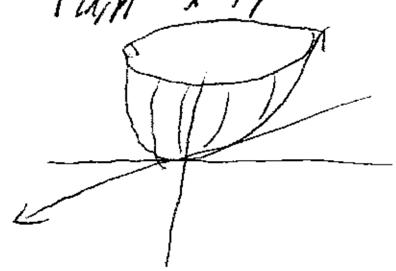
Example

$f(x) = x^2$



$\{(x,y) \mid y = x^2\}$

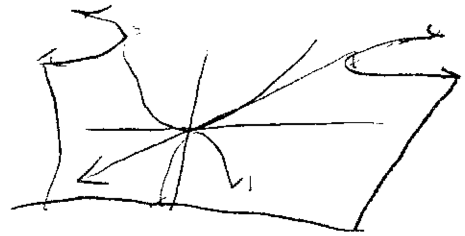
$f(x,y) = x^2 + y^2$



paraboloid

Example

$f(x,y) = x^2 - y^2$ Graph



saddle

Example

$f(x,y) = ax + by + c$ called a <u>linear function</u> of x & y

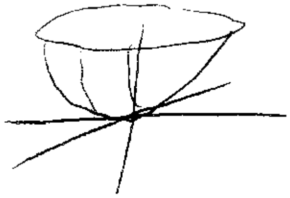
Graph is plane $z = ax + by + c$.

Level Curves

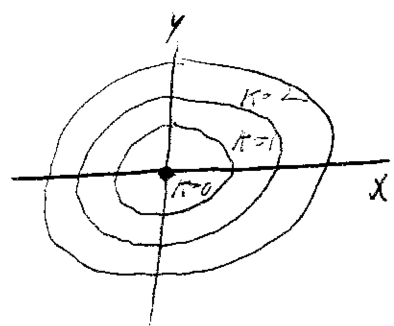
Graph $z = f(x,y)$ is just one way to represent a function. Another is to sketch <u>level curves</u> $f(x,y) = k$ for various choices of k.

Example

$f(x,y) = x^2 + y^2$



Graph

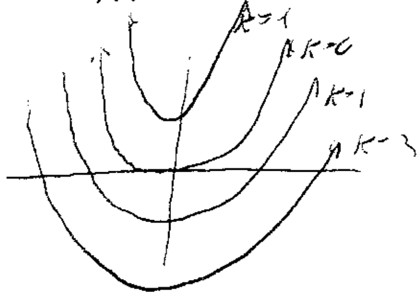


Sometimes called "contour map" or "isothermals" or "isobars"

f = Temp f = barom press.

Example

Let $f(x,y) = x^2 - y$. Sketch level curves



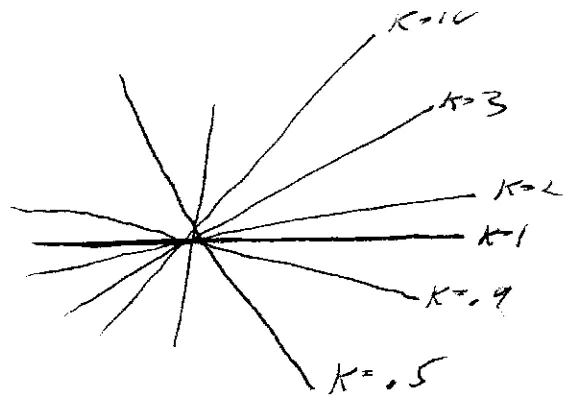
$k=0 \quad y = x^2$

Example

$f(x,y) = e^{y/x}$

$k = e^{y/x}$

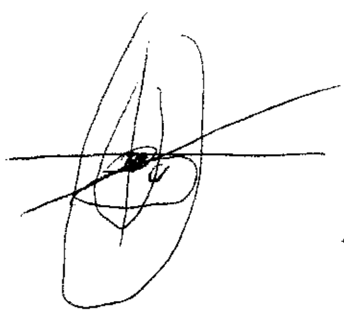
$\ln k = y/x \qquad y = x \ln k$



Computers are very helpful for plotting!

Example

$f(x,y,z) = x^2 + y^2 + \frac{z^2}{4}$ Sketch level surfaces



level surfaces are ellipsoids, long in z direct.

Rmk To graph this function would require 4-dimensions, i.e.

$w = x^2 + y^2 + z^2/4$

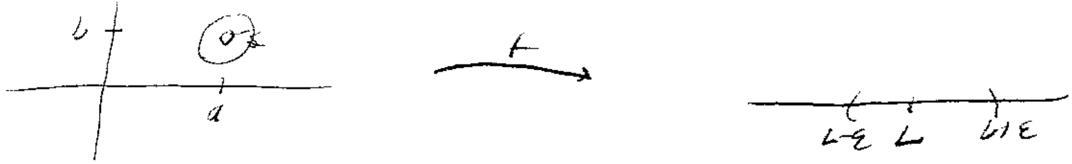
Example

$f(x,y) = \arcsin(x^2+y^2-2)$. Find and sketch the domain.

<u>Limits</u>

• [illegible]

Informal $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$



• Pick $\varepsilon > 0$ (think small)

• There is a $\delta > 0$ so the disk $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ maps into $(L-\varepsilon, L+\varepsilon)$

<u>Rmk</u> $f(a,b)$ is irrelevant

<u>Def</u> Let f be a function of two variables whose domain D includes points arbitrarily close to (a,b). Then

$$\lim_{(x,y)\to(a,b)} f(x,y) = L \quad \text{if}$$

For every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$(x,y) \in D \text{ and } 0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \quad \text{then} \quad |f(x,y) - L| < \varepsilon$$

<u>General def</u> $f(x_1, x_2, \ldots, x_n)$

$$\lim_{(x_1, x_2, \ldots, x_n) \to (a_1, a_2, \ldots, a_n)} f = L \text{ if } \ldots$$

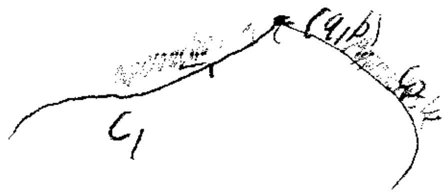
$$0 < \sqrt{(x_1 - a_1)^2 + \cdots + (x_n - a_n)^2} < \ldots$$

Useful idea

Suppose $f(x,y) \to L_1$ as $(x,y) \to (a,b)$ along path C_1 and
Suppose $f(x,y) \to L_2$ " " " " " C_2 with $L_1 \neq L_2$

Then $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ DNE.

Picture



Ex $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$

Approach on $y=0$, $x=0$, $y=x$

Ex $f(x,y) = \dfrac{xy^2}{x^2+y^4}$ $\lim\limits_{(x,y)\to(0,0)} f(x,y) = ??$

- $y = mx$ all $\to 0$
- parabola $x = y^2$

Limits are complicated!