Lecture 17

Review Triple integrals typically of form:

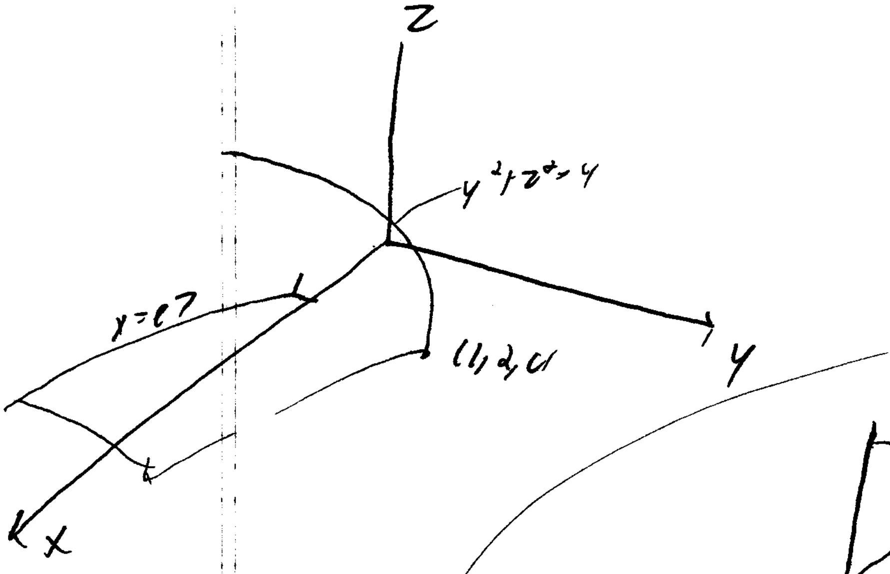
$$\int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\, dz\, dy\, dx$$

or 5 other variations.

Rmk Given region $E \subseteq \mathbb{R}^3$, $\text{Volume}(E) = \iiint_E 1\, dV$

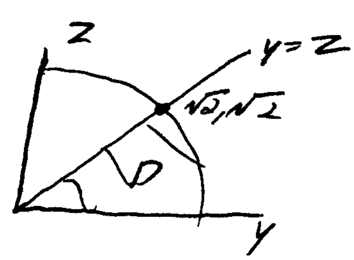
Ex Sketch solid whose volume is given by $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$.

Ex Evaluate $\iiint_E \frac{1}{x}\, dV$ E is 1^{st} octant region bounded by plane $x=1$, below by cylinder $x=e^z$ and above by plane $y=z$ and cylinder $y^2+z^2=4$.



- In front of $x=1$ and behind $x=e^z$ so

$$\iint_D \int_1^{e^x} \frac{1}{x}\, dx\, dA$$



$0 \le z \le \sqrt{2}$

$z \le y \le \sqrt{4-z^2}$

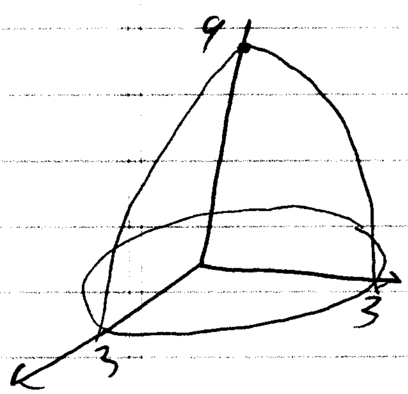
$$\int_0^{\sqrt{2}} \int_z^{\sqrt{4-z^2}} \int_1^{e^z} \frac{1}{x}\, dx\, dy\, dz$$

More changes of Coordinates

I. Cylindrical Just polar plus one extra directn

$x = r\cos\theta \quad y = r\sin\theta \quad z = z \qquad dV = r\,dr\,d\theta\,dz$

Ex Find volume bounded by $x^2+y^2+z=9$ and xy plane



cylindrical $0 \le \theta \le 2\pi$

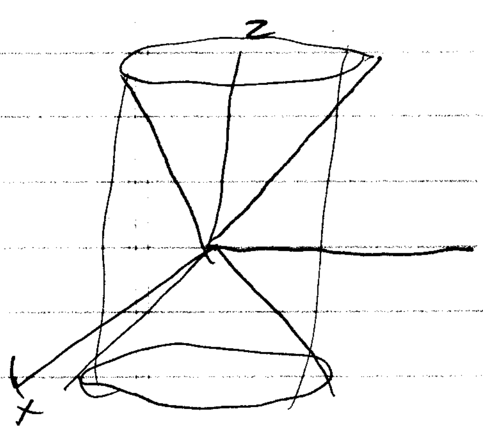
$0 \le r \le 3$

$0 \le z \le 9$

$$\int_0^9 \int_0^{2\pi} \int_0^3 (9-r^2)\, r\,dr\,d\theta\,dz$$

Ex Find volume of the solid generated by revolving the region in the xz plane bounded by $z=2x^2$, x axis and $x=1$ about the z-axis

Ex Find $\iiint_U z^4\,dV$ where U is bounded by cones $z=\sqrt{x^2+y^2}$, $z=-\sqrt{x^2+y^2}$, cylinder $x^2+y^2=1$, in back by $x=0$ to left by $y=0$



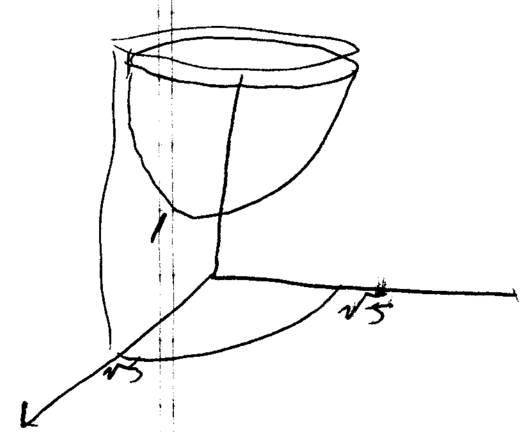
cylindrical: $0 \le \theta \le \pi/2$

$0 \le r \le 1$

$-r \le z \le r$

$$\int_0^{\pi/2} \int_0^1 \int_{-r}^{r} z^4\, r\,dz\,dr\,d\theta$$

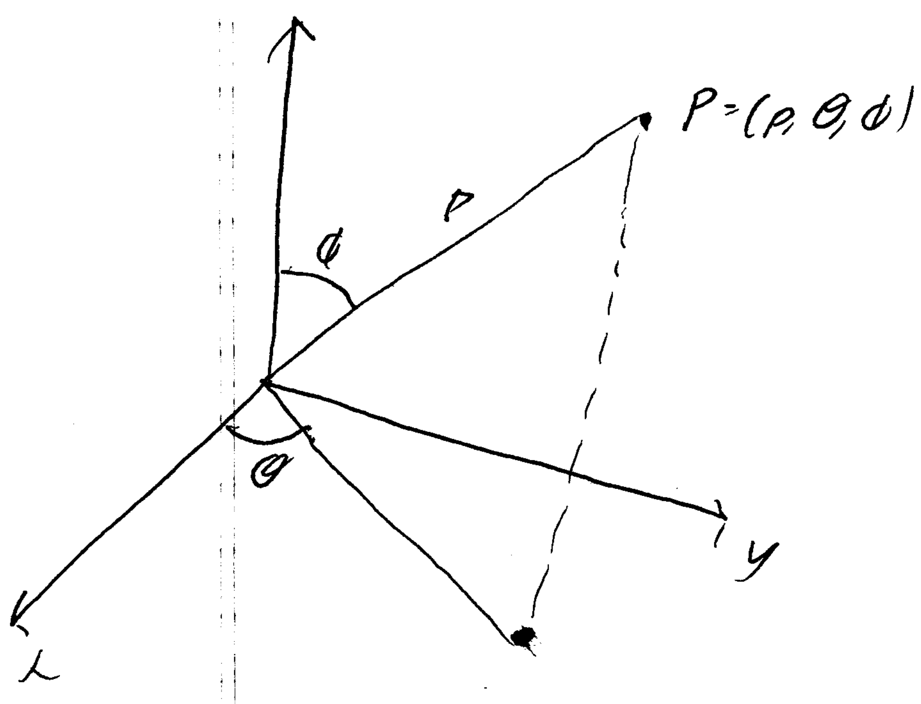
Ex Evaluate $\iiint_E e^z\,dV$ E bounded by paraboloid $z=1+x^2+y^2$ cylinder $x^2+y^2=5$ and xy plane



$0 \le \theta \le 2\pi,\ 0 \le r \le \sqrt{5},\ 0 \le z \le 1+r^2$

$$\int_0^{2\pi}\int_0^{\sqrt{5}}\int_0^{1+r^2} e^z\, r\,dz\,dr\,d\theta$$

SPHERICAL COORDINATES



$\rho \ge 0$ = dist from origin
θ = ∡ from pos x axis
ϕ = ∡ down from pos z axis

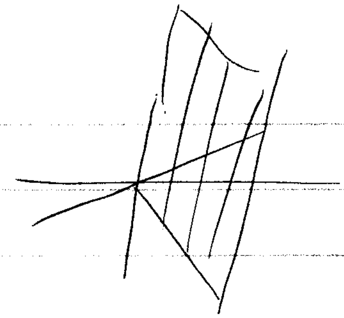
so $0 \le \theta \le 2\pi$
$0 \le \phi \le \pi$

$x = \rho \sin\phi \cos\theta \quad y = \rho \sin\phi \sin\theta \quad z = \rho\cos\phi$

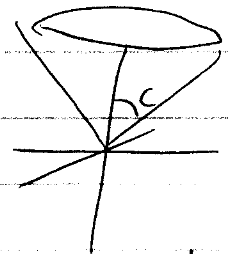
$\rho^2 = x^2+y^2+z^2$

<u>Some equations</u>

$\theta = c$

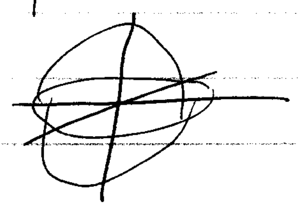


$\phi = c$



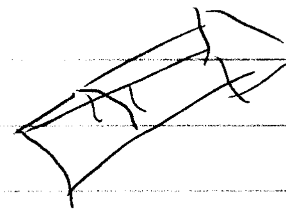
cone

$\rho = c$



sphere

"Spherical rect box"



$$\Delta V = \rho^2 \sin\phi \, \Delta\rho \, \Delta\phi \, \Delta\theta$$

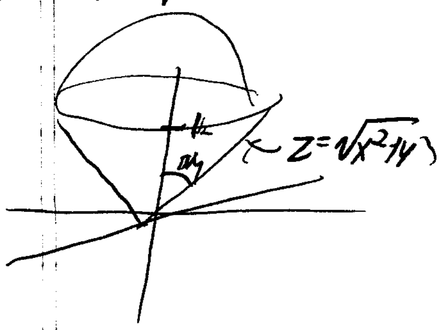
<u>Thm</u> $$\iiint_U f(x,y,z)\, dx\, dy\, dz = \iiint_{U \text{ in spher}} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

<u>Ex</u> Volume of sphere radius a = $\iiint_{\text{sphere}} 1 \, dV$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

Ex Find volume of solid above $z=\sqrt{x^2+y^2}$ below $x^2+y^2+z^2=z$

A. $x^2+y^2+z^2-z=0 \Rightarrow (x^2)+(y^2)+(z-\frac{1}{2})^2=1$



CONE $\rho\cos\phi = \sqrt{\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta} = \rho\sin\phi$

so $\phi=\pi/4$

$0\le\theta\le 2\pi$

SPHERE $\rho^2-\rho\cos\phi=0$

$\rho=\cos\phi$

$0\le\theta\le 2\pi$

$0\le\phi\le\pi/4$

$0\le\rho\le\cos\phi$

Ex $\iiint_E z\,dV$ E in 1^{st} oct betw spheres $x^2+y^2+z^2=1$

$x^2+y^2+z^2=4$