# Math 241 Lecture 1

## I. Intro

- text, webpage, online HW, used books, calculators, Lab, quizzes

## II. Multivariable vs Single Variable

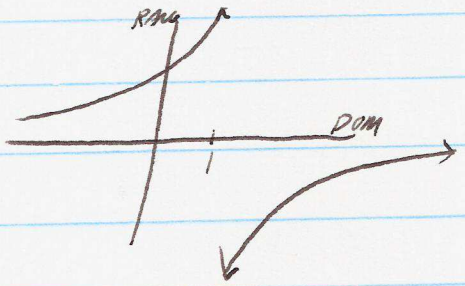
### Single:

Functions have domain $\subseteq \mathbb{R}$, range $\subseteq \mathbb{R}$

Ex $f(x) = \frac{1}{1-x}$ Domain $(-\infty, 1) \cup (1, \infty)$

Range $= (-\infty, 0) \cup (0, \infty)$

Graph




Points on graph are $(x, f(x))$

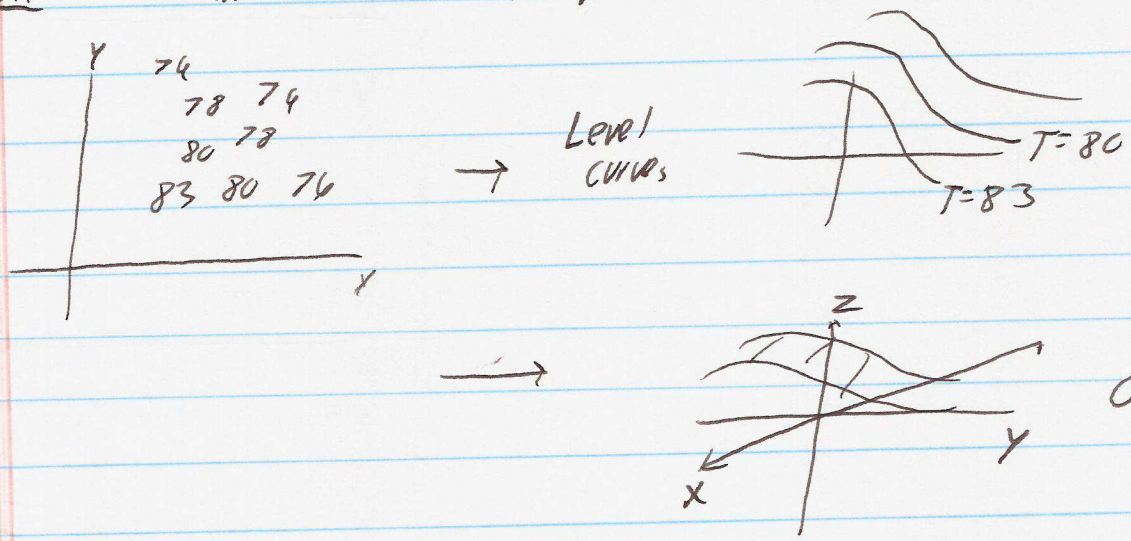
Ex $h(t) = -\frac{1}{2}t^2 + t + 1$ height of ball

Derivative $h'(t) = \frac{dh}{dt}$ measures rate of change of $h(t)$ with respect to $t$

Goal: Understand things like graphs, limits, derivatives, continuity, etc.. for more complicated functions,

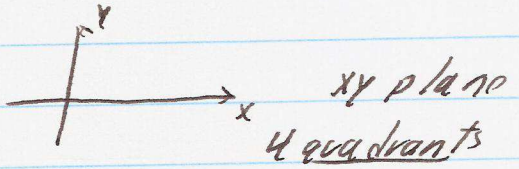
i.e. w/ more than one variable of input and/or output.

Example $T: \mathbb{R}^2 \to \mathbb{R}$ $T(x,y)$ = temp at $(x,y)$

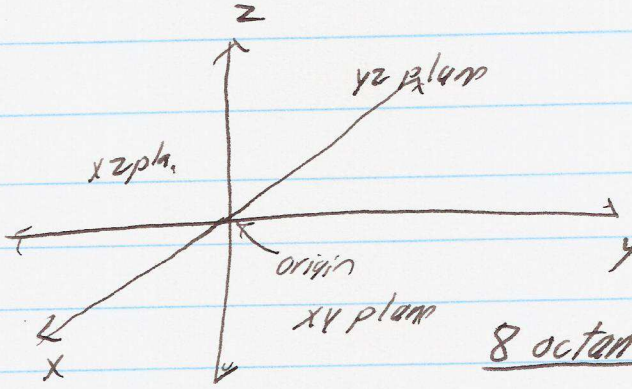



## III. 3-D coordinates

$\mathbb{R}^2 = \{(a,b) \mid a,b \in \mathbb{R}\}$



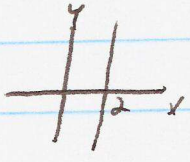

$\mathbb{R}^3 = \{(a,b,c) \mid a,b,c \in \mathbb{R}\}$



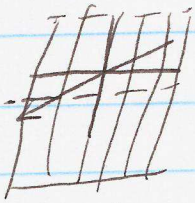

point has 3 coordinates

GRAPHS

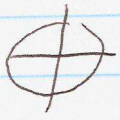
2D: $x=2$



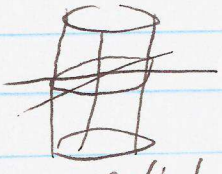

3D



2D $x^2+y^2=1$



3D



cylinder

Distance Formula

Let $P_1 = (x_1, y_1, z_1)$ $P_2 = (x_2, y_2, z_2)$

$$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Ex Find distance $(1,2,-5)$ to $(2,9,1)$

$\sqrt{1^2+2^2+6^2} = \sqrt{41}$

Proof Put Thm

Application

Fix a point $(h,k,l)$. The distance from $(x,y,z)$ to $(h,k,l)$ is

$$\sqrt{(x-h)^2+(y-k)^2+(z-l)^2}$$

* Equation of a sphere radius r, center $(h,k,l)$ is

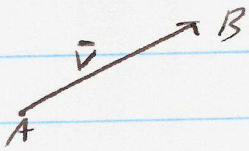
$$(x-h)^2+(y-k)^2+(z-r)^2$$

Ex 1. $x^2+y^2+z^2-6x+4y-2z=11$ Find cent & radius

2. Sketch region $4 \le x^2+y^2+z^2 \le 9$

# Vectors

Def Vector quantity has magnitude & directions




$\vec{V} = \overrightarrow{AB}$

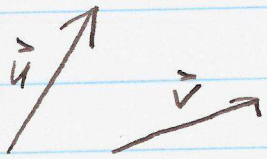
"$\vec{0}$ vector" no direction

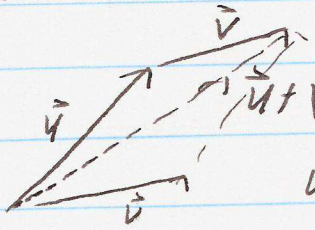
Example velocity, momentum

nonexample weight

## Vector Operations

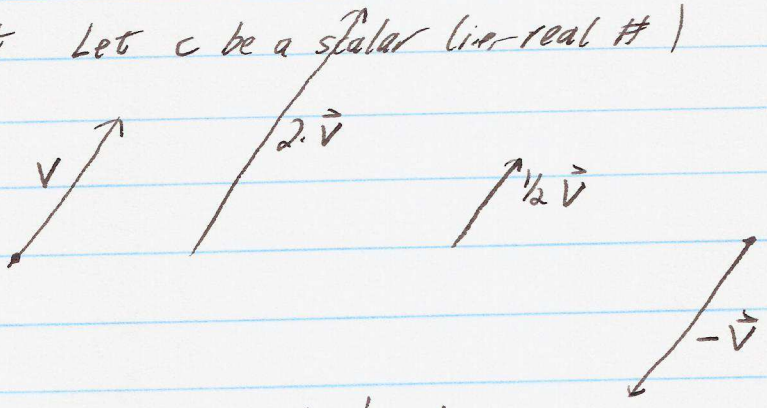
Addition put initial pt on $\vec{v}$ at end of $\vec{u}$



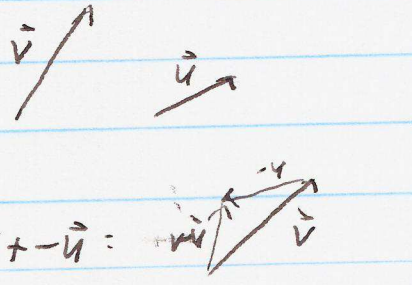

$\vec{u}+\vec{v} = \vec{v}+\vec{u}$

Scalar Mult Let c be a scalar (i.e. real #)



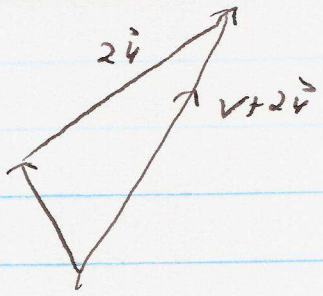

* neg. $\vec{v}$ pts in opposite direction

Example

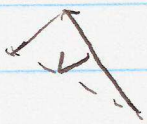


Ex 


$v+2\vec{u}$

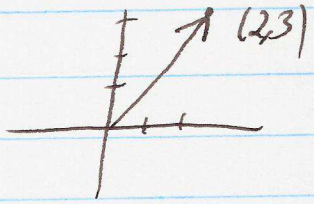



$v-\frac{1}{2}u$



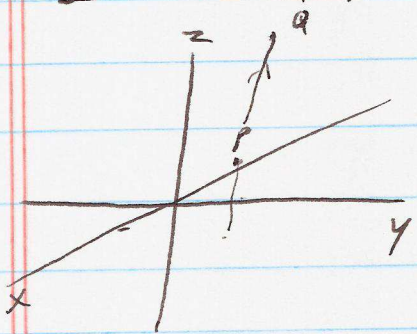

Use coordinates


Put initial point at origin




Fact Let $P=(x_1,y_1,z_1)$ $Q=(x_2,y_2,z_2)$ The vector $\vec{PQ}$ has coordinates $(x_2-x_1, y_2-y_1, z_2-z_1)$

Ex $P=(1,2,1)$ $Q=(0,2,4)$

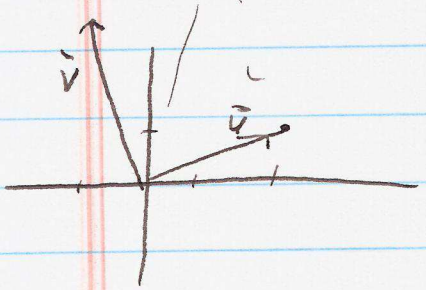



$\vec{PQ}=\langle -1,0,3\rangle$

* Add vectors $\longleftrightarrow$ add coordinates
* Subtract vectors $\longleftrightarrow$ subtract coordinates
* scalar mult $\longleftrightarrow$ mult coor

Ex

$\vec{u} = \langle 2, 1 \rangle$ $\vec{v} = \langle -1, 3 \rangle$




$\vec{u} + \vec{v} = \langle 1, 4 \rangle$

$\frac{1}{2}\vec{u} = \langle 1, \frac{1}{2} \rangle$

$\vec{u} - 2\vec{v} = \langle 4, -5 \rangle$

etc.