Name: SOLUTIONS

Quiz #7 - March 17, 2009

1. Let $f(x) = \frac{3}{64}x\sqrt{16-x^2}$ for $0 \le x \le 4$ and $f(x) = 0$ for all other values of x. Verify that $f(x)$ is a probability density function and find $P(X < 2)$.

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^4 \frac{3}{64} x\sqrt{16-x^2}\,dx \qquad u = 16-x^2 \quad du = -2x\,dx$$

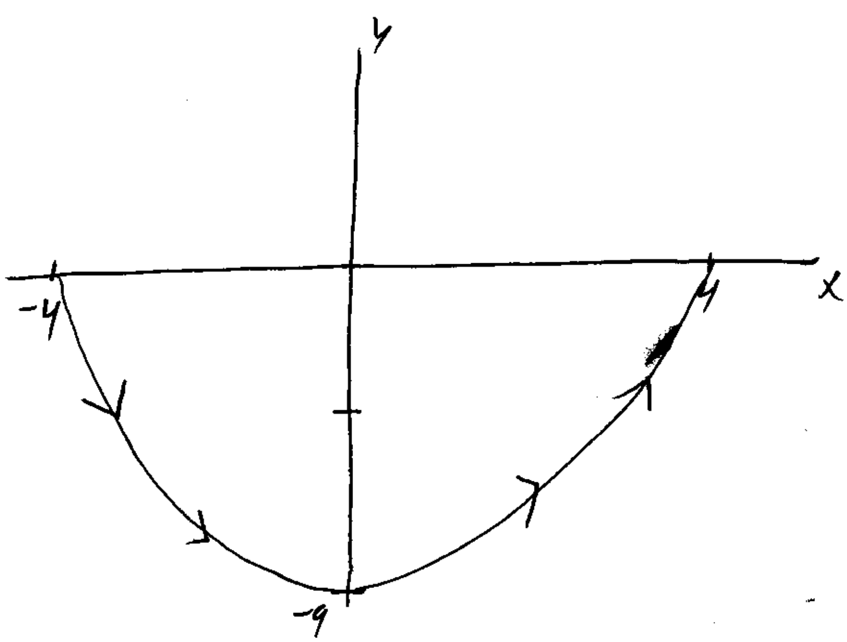
$$= \int \frac{-3}{128}\sqrt{u}\,du = -\frac{1}{64}(16-x^2)^{3/2}\Big|_0^4$$

$$= -\frac{1}{64}(0-64) = 1 \quad \text{so P.D.F. } \checkmark$$

$$P(X<2) = \int_0^2 f(x)\,dx = -\frac{1}{64}(16-x^2)^{3/2}\Big|_0^2$$

$$= -\frac{1}{64}\left(12^{3/2} - 64\right)$$

2. Let $x = 4\cos t$, $y = 9\sin t$ for $\pi \le t \le 2\pi$. Sketch the curve neatly (axes clearly labeled, etc..), indicating with an arrow the direction in which the curve is traced as the parameter increases.



Name: SOLUTIONS

Quiz #7 - March 19, 2009

1. Let $x = 10 - t^2$, $y = t^3 - 12t$. Find the points on the curve where the tangent line is horizontal or vertical..

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-12}{-2t}$$

Horizontal : $3t^2-12=0$ $t = \pm 2 \rightarrow \boxed{(6,-16), (6,16)}$

Vertical: $-2t=0$ $t=0$ $\boxed{(10,0)}$

2. Let $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \le t \le 1$. Find the exact length of the curve. Hint for integral: $\sqrt{t^2} = t$ for $t \ge 0$.

$$L = \int_0^1 \sqrt{x'(t)^2 + y'(t)^2}\, dt = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{36t^2 + 36t^4}\, dt$$

$$= \int_0^1 6t\sqrt{1+t^2}\, dt$$

$u = 1+t^2$ $du = 2t\,dt$

$$= \int_1^2 3u^{1/2}\,du = 2u^{3/2}\Big|_1^2$$

$$= 2(2^{3/2} - 1)$$