Quadratic Quiz #2 - January 29, 2009

1. Let \( f(x) = \sqrt{x} \) on \([0, 4]\). Find the average value of \( f(x) \) on this interval. Then find \( c \) in \([0, 4]\) such that \( f_{\text{ave}} = f(c) \).

\[
\int_0^4 f(x) \, dx = \frac{1}{4} \int_0^4 \sqrt{x} \, dx = \frac{1}{4} \left[ \frac{2}{3} x^{3/2} \right]_0^4 = \frac{1}{6} (4^{3/2} - 0) = \frac{8}{9} = \left( \sqrt[3]{4} \right)
\]

\[
\frac{y}{3} = \sqrt{x} \quad \frac{16}{9} = x
\]

\[
C = \frac{16}{9}
\]

2. Calculate

\[
\int te^{\frac{t}{2}} \, dt.
\]

\[
u = t, \quad v = 2e^{\frac{t}{2}}
\]

\[
dv = dt, \quad du = e^{\frac{t}{2}}
\]

\[
\int te^{\frac{t}{2}} \, dt = 2te^{\frac{t}{2}} - 2e^{\frac{t}{2}}
\]

\[
= 2te^{\frac{t}{2}} - 4e^{\frac{t}{2}} + C
\]
Quiz #2 - January 27, 2009

1. Consider the region under the graph of \( y = \sin(x^2) \) and above the \( x \) axis for \( 0 \leq x \leq \sqrt{\pi} \). Use the method of cylindrical shells to find the volume of the solid obtained by rotating this region around the \( y \) axis.

\[
V = \int_{0}^{\sqrt{\pi}} 2\pi x \sin(x^2) \, dx
\]

Let \( u = x^2 \), \( du = 2x \, dx \)

\[
V = \int_{0}^{\pi} \pi \sin(u) \, du = \pi [ -\cos(u) ]_{0}^{\pi} = \pi ( -1 - 1 ) = 2\pi
\]

2. Find the average value of the function \( f(x) = x + x^2 \) on the interval \([0, 2]\).

\[
\text{Average Value} = \frac{1}{2} \left( \int_{0}^{2} x + x^2 \, dx \right) = \frac{1}{2} \left[ \frac{x^2}{2} + \frac{x^3}{3} \right]_{0}^{2} = \frac{1}{2} \left( 2 + \frac{8}{3} \right) = \frac{1 + 8/3}{2} = \frac{11}{6}
\]