

Name:

# SOLUTIONS

Quiz #4 - February 10, 2009

1. Compute

$$\int \frac{x+1}{(x-2)^2} dx.$$

$$\frac{x+1}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$$\int \frac{1}{x-2} + \frac{3}{(x-2)^2} dx$$

$$x+1 = A(x-2) + B$$

$$x=2 \Rightarrow B=3$$

$$x=0 \Rightarrow 1 = -2A + B$$
$$A=1$$

$$= \boxed{\ln|x-2| - \frac{3}{x-2} + C}$$

2. Compute

$$\int \frac{x-1}{4x^2-4x+3} dx.$$

$$4x^2-4x+3 = (2x-1)^2 + 2$$

$$\text{Let } u=2x-1 \quad du=2dx$$
$$x = \frac{1}{2}u + \frac{1}{2}$$

$$\int \frac{\frac{1}{2}u - \frac{1}{2}}{u^2+2} \cdot \frac{1}{2} du = \frac{1}{4} \int \frac{u-1}{u^2+2} du$$

$$= \frac{1}{4} \int \frac{u}{u^2+2} du - \frac{1}{4} \int \frac{1}{u^2+2} du$$

$$= \frac{1}{8} \ln|u^2+2| - \frac{1}{4} \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$= \frac{1}{8} \ln|4x^2-4x+3| - \frac{1}{4\sqrt{2}} \tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$$

Name:

## SOLUTIONS

Quiz #4 - February 12, 2009

1. Compute

$$\int \frac{x^3 + 4}{x^2 + 4x + 4} dx.$$

$$\begin{array}{r} x-4 \\ x^2+4x+4 \overline{) x^3+4} \\ \underline{x^3+4x^2+4x} \phantom{+4} \\ -4x^2-4x+4 \\ \underline{-4x^2-16x-16} \\ 12x+20 \end{array}$$

$$= \int x-4 + \frac{12x+20}{x^2+4x+4} dx$$

$$= \frac{x^2}{2} - 4x + 4 \int \frac{3x+5}{(x+2)^2}$$

$$\frac{3x+5}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

$$3x+5 = A(x+2) + B$$

$$x=-2 \quad -1 = B$$

$$x=0 \quad 5 = 2A + B \quad A = 3$$

$$\frac{x^2}{2} - 4x + 12 \ln|x+2| + \frac{4}{x+2} + C$$

2. Compute

$$\int \frac{1}{(x-2)(x+1)} dx.$$

$$\frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$A(x+1) + B(x-2) = 1$$

$$x=-1 \Rightarrow B = -1/3$$

$$x=2 \Rightarrow A = 1/3$$

$$\frac{1}{3} \ln|x-2| - \frac{1}{3} \ln|x+1| + C$$