

Lecture 8 Integration using Tables

Review u -substitution: Goal is to put an integral into form on our "list".

$$\text{Ex } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\text{Proof } x = a \tan \theta \quad dx = a \sec^2 \theta d\theta \rightarrow \int \frac{a \sec^2 \theta d\theta}{a^2 \sec^2 \theta} = \int \frac{1}{a} d\theta = \frac{\theta}{a}$$

$$\text{But } \theta = \tan^{-1}\left(\frac{x}{a}\right) \quad //$$

Work is done, why repeat.

$$\text{Ex } \int \frac{e^x}{e^{2x}+9} dx \quad u = e^x \quad du = e^x dx$$

$$= \int \frac{du}{u^2+9} = \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) + C$$

$$= \frac{1}{3} \tan^{-1}\left(\frac{e^x}{3}\right) + C$$

Tables are just lists of formulas like $\int \frac{1}{u^2+a^2} du = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$

You need to recognize which formula to use, keeping in mind that a substitution is usually required to make your integral match the list.

$$\text{Ex } \int_2^3 \frac{1}{x^2 \sqrt{4x^2-7}} \quad \# 45: \int \frac{du}{u^2 \sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2 u} + C$$

$$\left(\begin{array}{l} u = 2x \quad a = \sqrt{7} \\ = 4 \int \frac{du}{u^2 \sqrt{u^2-a^2}} = \frac{4 \sqrt{u^2-a^2}}{a^2 u} = \frac{4 \sqrt{4x^2-7}}{14x} \Big|_2^3 = \frac{4\sqrt{29}}{42} - \frac{4\sqrt{9}}{28} \end{array} \right)$$

Ex $\int \frac{\tan^3(1/2)}{2^2}$ # 69: $\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln|\cos u| + C$

$u = 1/2 \quad du = -1/2 \, dz$

$= -\frac{1}{2} \tan^2(1/2) - \ln|\cos(1/2)| + C$

Ex $\int x \sqrt{3x^2 + x + 1} \, dx$ Formulas 21-29 "Forms involving $\sqrt{a^2 + u^2}$ "

$3x^2 + x + 1 = (\sqrt{3}x + \frac{1}{2\sqrt{3}}) \sqrt{3x + \frac{1}{2\sqrt{3}}} + 11/12$

$= \int x \sqrt{(\sqrt{3}x + \frac{1}{2\sqrt{3}})^2 + 11/12}$

$u = \sqrt{3}x + \frac{1}{2\sqrt{3}} \quad du = \sqrt{3} \, dx$

$x = \sqrt{3}u - 1/2$

Formulas

$= \int (\sqrt{3}u - 1/2) \sqrt{u^2 + 11/12} \, du$

$= \int \sqrt{3}u \sqrt{u^2 + 11/12} \, du - \frac{1}{2} \int \sqrt{u^2 + 11/12} \, du$

$v = u^2 + 11/12$
 $dv = 2u \, du$

$a = \sqrt{11/12}$

#21 $\int \sqrt{a^2 + u^2} \, du$

$= \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln \left| \frac{u}{a} + \sqrt{1 + \frac{u^2}{a^2}} \right|$

$= \frac{\sqrt{3}}{3} (u^2 + 11/12)^{3/2} - \frac{1}{2} \left(\frac{u}{2} \sqrt{u^2 + 11/12} + \frac{11}{24} \ln \left| u + \sqrt{u^2 + 11/12} \right| \right)$

$= \frac{\sqrt{3}}{3} (3x^2 + x + 1)^{3/2} - \frac{\sqrt{3}x + \frac{1}{2\sqrt{3}}}{4} \sqrt{3x^2 + x + 1} - \frac{11}{48} \ln \left| \sqrt{3}x + \frac{1}{2\sqrt{3}} + \sqrt{3x^2 + x + 1} \right|$

Computer Algebra

MAPLE $\int x^2 dx \rightarrow \text{int}(x^2, x);$

$\int 3\sin(x^2) dx \rightarrow \text{int}(3*\sin(x^2), x);$

$\int_1^3 x^2 + 1 dx \rightarrow \text{int}(x^2 + 1, x=1..3);$

Numerical approx

$\text{evalf}(\text{int}(e^{x^2}, x=1..3));$ 1443082472

$\int_1^3 e^{x^2} dx$

Remark

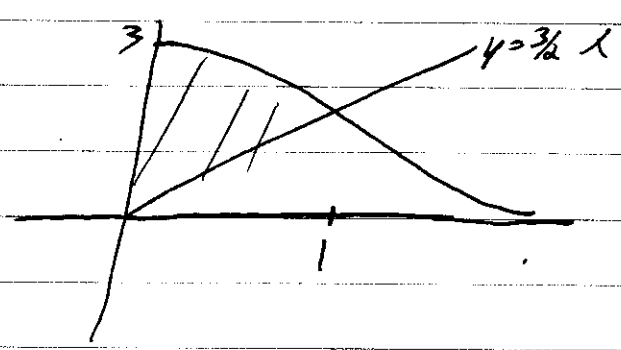
There is no nice formula for $\int e^{x^2} dx$ but

FTOC says $g(t) = \int_1^t e^{x^2} dx$ is a nice continuous function with $g'(t) = e^t$.

Review

CHPT 6 Area between Curves

Ex Find area between $y = \frac{3}{x^2+1}$ and $y = \frac{3}{2}x$ with $x \geq 0$



$$\frac{3}{2}x = \frac{3}{x^2+1}$$

$$x=1$$

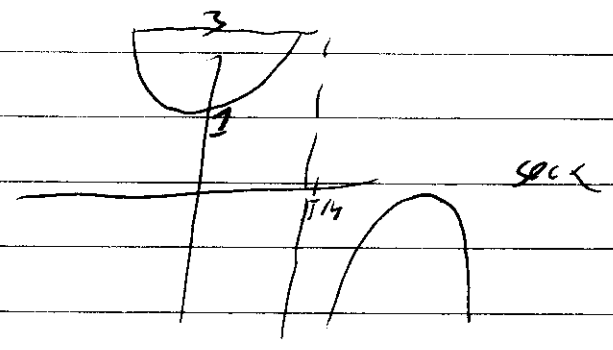
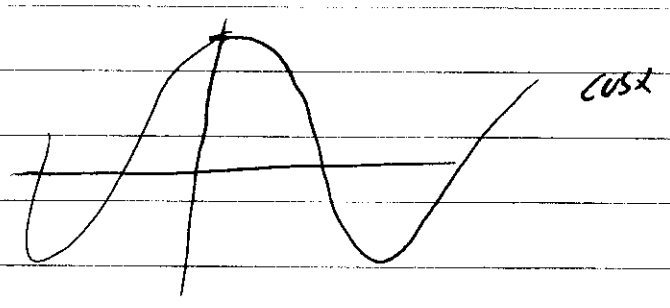
$$\int_0^1 \left(\frac{3}{x^2+1} - \frac{3}{2}x \right) dx = 3 \tan^{-1} x - \frac{3}{4} x^2 \Big|_0^1$$

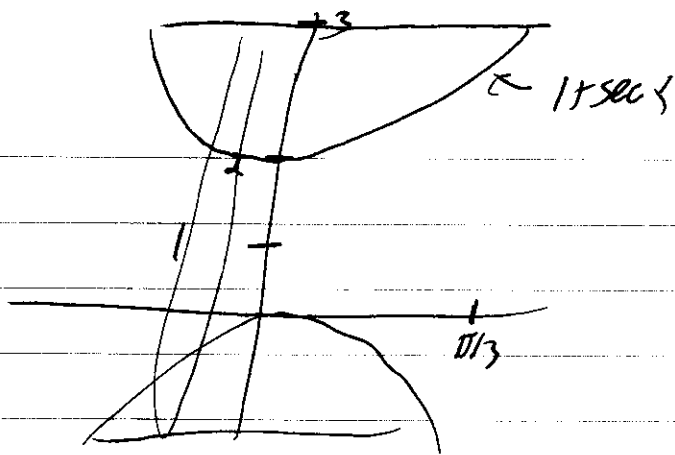
$$= \left(3 \frac{\pi}{4} - \frac{3}{4} \right) - (0)$$

$$= \frac{3}{4} (\pi - 1)$$

Ex Volumes $V = \int A(x) dx$

Find volume rotating $y = 1 + \sec x$, $y = 3$ about $y = 1$.





$$1 + \sec x = 3$$

$$\sec x = 2$$

$$\cos x = 1/2 \quad x = \pi/3, -\pi/3$$

$$V = \int_{-\pi/3}^{\pi/3} \pi (2^2) - \pi (\sec^2 x) dx$$

$$= \int_{-\pi/3}^{\pi/3} 4\pi - \pi \sec^2 x dx$$

$$= 4\pi x - \pi \tan x \Big|_{-\pi/3}^{\pi/3}$$

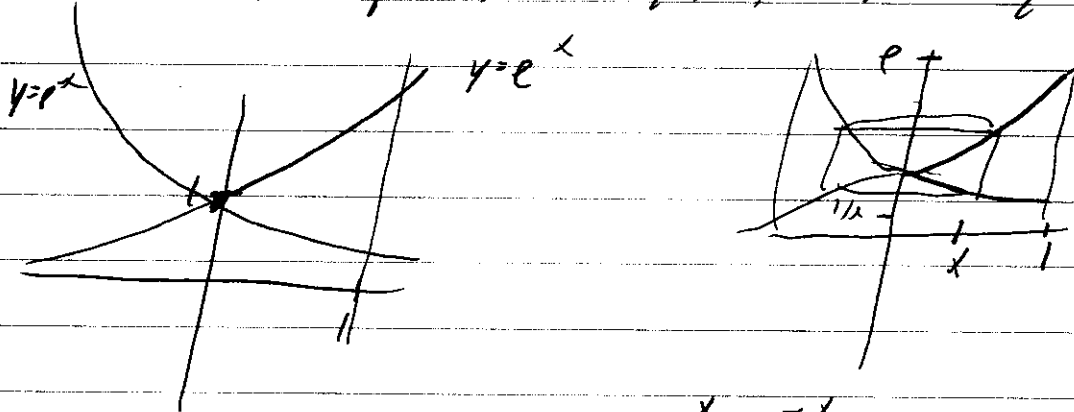
$$= \left(\frac{4}{3}\pi^2 - \pi \frac{\sqrt{3}}{2} \right) - \left(-\frac{4}{3}\pi^2 + \pi \frac{\sqrt{3}}{2} \right)$$

$$= \frac{8}{3}\pi^2 - \pi\sqrt{3}$$

Shells

1. Sketch, determine which way to integrate
2. Draw arbitrary shell, find h, r in terms of variable from #1.
3. $V = \int 2\pi r h \Delta V$

Ex Find volume region $y=e^x$, $y=e^{-x}$, $x=1$ about y axis



Shell: $R=x$ $H=e^x - e^{-x}$

$$V = \int_0^1 2\pi x (e^x - e^{-x}) dx$$

IBP!