Lecture 11

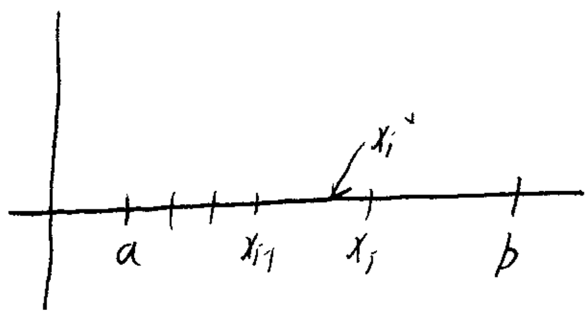
- go over exam

Approximate / Numerical Integration

Review

Integral as limit of Riemann Sums

Problem Suppose computer approximating, only finitely many terms, what is the error?



$\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$, $x_i^* \in [x_{i-1}, x_i]$

Example:

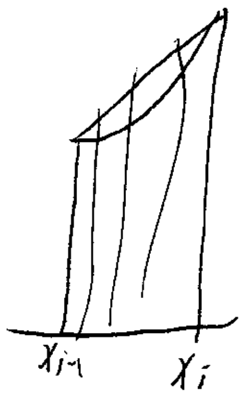
using left endpoint $L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x$

right endpoint $R_n = \sum_{i=1}^{n} f(x_i)\Delta x$

Midpoint rule: $x_i^* = \frac{x_{i-1}+x_i}{2}$ $M_n = \sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x$

Trapezoidal Rule

Recall Area of a trapezoid = base · Avg height



$$\text{Area} = \Delta x \left(\frac{f(x_{i-1}) + f(x_i)}{2} \right)$$

$$T_n = \Delta x \left(\frac{f(x_0)+f(x_1)}{2} + \frac{f(x_1)+f(x_2)}{2} + \dots + \frac{f(x_{n-1})+f(x_n)}{2} \right)$$

$$= \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n) \right)$$

Thm $\int_a^b f(x)\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n = \lim_{n\to\infty} T_n = \lim_{n\to\infty} M_n$

but diff't approximations may give better estimates

Theorem Suppose $|f''(x)| \le K$ $\forall\ a \le x \le b$. Let E_T and E_M be error w/ Trap & Midpt Rule. Then

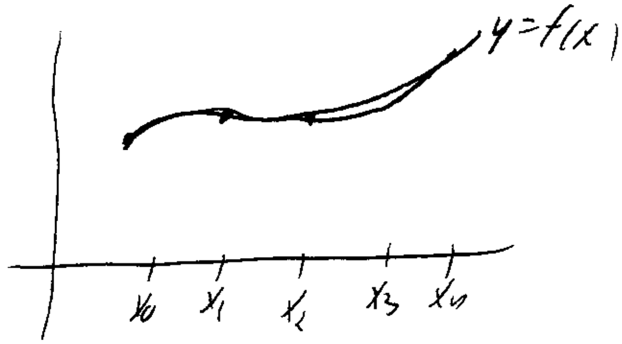
$$|E_T| \le \frac{K(b-a)^3}{12n^2} \qquad |E_M| \le \frac{K(b-a)^3}{24n^2}$$

Rmk Does not mean trap rule is always better.

Simpson's Rule

Fact: 3 points give a unique parabola

- use even # of intervals



- put parabolas
- calculate area

$$S_n = \frac{\Delta x}{3}\left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\dots+2f(x_{n-2}) + 4f(x_{n-1})+f(x_n)\right)$$

Thm: Suppose $f''''(x) \le K \ \forall a \le x \le b$. Then

$$|E_S| \le \frac{K(b-a)^5}{180n^4}$$

Problems

1. $\int_0^4 e^{\sqrt{t}} \sin t \, dt \quad n=8$

   - Do midpoint, trapezoid, Simpson and get error bounds

2. #21 p.505

#3. Guarantee Trap rule for $\int_1^2 1/x \, dx$ w/in .001. How many n?