Lecture 1

- go over syllabus, labs, Webassign, calculators

Antiderivatives

Recall: $\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$ indefinite integral.

Ex $\int x^2\,dx = \frac{x^3}{3} + C$

Definite Integral: $\int_1^3 x^2\,dx = \left.\frac{x^3}{3}\right|_1^3 = 9 - 1/3 = 26/3$

Differentiation Rules and Formulas give corresponding rules for integrals:

Formulas

1. $\int k\,du = ku + C$
2. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1$
3. $\int \frac{1}{u}\,du = \ln|u| + C$
4. $\int a^u\,du = \frac{a^u}{\ln a} + C$
5. $\int e^u\,du = e^u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sin u\,du = -\cos u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \frac{1}{1+u^2}\,du = \tan^{-1} u + C$
13. $\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1} u + C$
14. $\int \sinh u\,du = \cosh u + C$

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Rules

1. $\int c \cdot f(u)\,du = c\int f(u)\,du$ 2. $\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du$

Goal Uses these formulas to compute many more antiderivatives.

Step 1 Learn 1-14, 1-2 cold!

Step 2 Recall if $u = g(x)$ then $du = g'(x)\,dx$ (differential)

Also $\frac{d}{dx}(f(u)) = f'(g(x)) \cdot g'(x)\,dx$

Example

1. $\int 2t\cos(t^2)\,dt$ Rmk $2t\cos(t^2)$ not on list!

$$\text{Let } u = t^2 \quad du = 2t\,dt$$

$$\int 2t\cos(t^2)\,dt = \int \cos u\,du = \sin u + C = \sin(t^2) + C$$

2. $\int x e^{x^2+1}\,dx$ $u = x^2+1$ $du = 2x\,dx$

So $x\,dx = \frac{1}{2}\,du$

$$\int x e^{x^2+1}\,dx = \int \tfrac{1}{2} e^u\,du = \tfrac{1}{2}\int e^u\,du = \tfrac{1}{2}e^u + C$$

$$= \tfrac{1}{2}e^{x^2+1} + C$$

Procedure for "u-substitution"

1. Choose u - goal is to simplify $\int$, get to known list
2. Compute du
3. Replace all "x" terms with "u" terms and integrate
4. Replace "u"'s with "x"'s

More examples

3. $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx \qquad u=\cos x \quad du=-\sin x\,dx$

$$= \int -\frac{1}{u}\,du = -\ln|u|+C = -\ln|\cos x|+C$$
$$= \ln\left|\frac{1}{\cos x}\right|+C$$
$$= \ln|\sec x|+C$$

$$\boxed{\int \tan x\,dx = \ln|\sec x|+C}$$

Also

$$\int \cot x\,dx = \ln|\sin x|+C = -\ln|\csc x|+C$$

4. $\int \sqrt{1+x^2}\;x^5\,dx$

$$u=1+x^2 \quad du=2x\,dx \qquad \text{so } x^2=u-1$$

$\sqrt{1+x^2}\;x^4\;\underbrace{x\,dx}_{\frac{1}{2}du}$ — $\sqrt{1+x^2}\to\sqrt{u}$, $x^4\to(u-1)^2$ — extra work in step 3!

$$= \int \sqrt{u}\,(u-1)^2\,\tfrac{1}{2}\,du = \tfrac{1}{2}\int u^{1/2}(u^2-2u+1)\,du$$
$$= \tfrac{1}{2}\int u^{5/2}-2u^{3/2}+u^{1/2}\,du$$
$$= \tfrac{1}{2}\left(\tfrac{2}{7}u^{7/2}-\tfrac{4}{5}u^{5/2}+\tfrac{2}{3}u^{3/2}\right)+C$$
$$= \tfrac{1}{2}\left(\tfrac{2}{7}(1+x^2)^{7/2}-\tfrac{4}{5}(1+x^2)^{5/2}+\tfrac{2}{3}(1+x^2)^{3/2}\right)+C$$

5. $\int \cos(2x)\,dx \qquad u=2x \quad du=2\,dx$

$$\int \tfrac{1}{2}\cos u\,du = \tfrac{1}{2}\sin u+C = \tfrac{1}{2}\sin(2x)+C$$

u-substitution for definite integrals $\int_a^b f(x)\,dx$

Method 1 Ignore a, b. Do $\int f(x)\,dx$ using u-subs, get back in terms of x and plug in b, a.

Ex $\displaystyle\int_1^2 x e^{x^2+1}\,dx = \frac{1}{2}e^{x^2+1}\Big|_{x=1}^{x=2} = \frac{1}{2}(e^5 - e^2)$

Method 2 Change limits of integration $x=a, x=b$ to $u=\_,\_$
Skip "step 4," just plug in u limits

Ex $\displaystyle\int_1^2 x e^{x^2+1}\,dx$ $\quad u = x^2+1 \quad du = 2x\,dx$

$x=1 \rightarrow u=2$

$x=2 \rightarrow u=5$

$$= \int_2^5 \frac{1}{2}e^u\,du = \frac{1}{2}e^u\Big|_{u=2}^{u=5} = \frac{1}{2}(e^5 - e^2)$$

Rmk Either method is ok, same answer.

Ex $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx$ $\quad u = 1/x \quad du = -1/x^2\,dx$

$x=1 \rightarrow u=1$

$x=2 \rightarrow u=1/2$

$$\int_1^{1/2} -e^u\,du = -e^u\Big|_1^{1/2} = -(e^{1/2} - e)$$

$$= e - \sqrt{e}$$

<u>Application</u> Recall $f(x)$ is <u>odd</u> if $f(-x) = -f(x)$

<u>even</u> if $f(x) = f(-x)$

<u>Thm</u> Suppose f is continuous on $[-a, a]$.

1. f even then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

2. f odd then $\int_{-a}^{a} f(x)\,dx = 0$

<u>Proof</u> $\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$

$u = -x \quad du = -dx$

$$\int_{a}^{0} -f(-u)\,du = \int_{0}^{a} f(-u)\,du$$

$$= \begin{cases} \int_{0}^{a} -f(u)\,du & \text{if odd} \\ \int_{0}^{a} f(u)\,du & \text{if even} \end{cases} \quad //$$

<u>Problems</u>

1. ~~$\int \tan x$~~ dx $\quad \int x(x^2-5)^{15}\,dx$

2. $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

3. $\int \frac{e^x}{e^x+1}\,dx$

4. $\int \frac{x}{1+x^4}\,dx$

5. $\int_{0}^{1} \frac{e^z+1}{e^z+z}$