Lecture 9

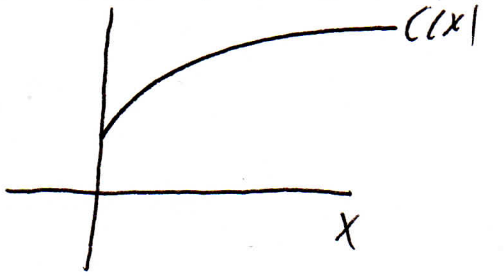
Review Avg rate of change of $f(x)$ from $x=x_1$ to $x=x_2$ is $\dfrac{f(x_2)-f(x_1)}{x_2-x_1} = \dfrac{\Delta y}{\Delta x}$ ← change in ← change in

Letting x_2 & x_1 get close we get:

$$f'(a) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$ derivative of f at a.

Interpretation 1. $f'(a)$ is instantaneous R.O.C of $f(x)$ wrt x at $x=a$.
2. $f'(a)$ is slope of tangent line.

Ex $C(x)$ = cost ($) to produce x widgets $C'(x)$ called marginal cost



* declining marginal cost
* economy of scale

Ex Let $g(x)=x^3+2x-1$ Find $g'(x)$ from the def. Find tang line at $x=2$.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3+2(x+h)-1 \;-(x^3+2x-1)}{h}$$

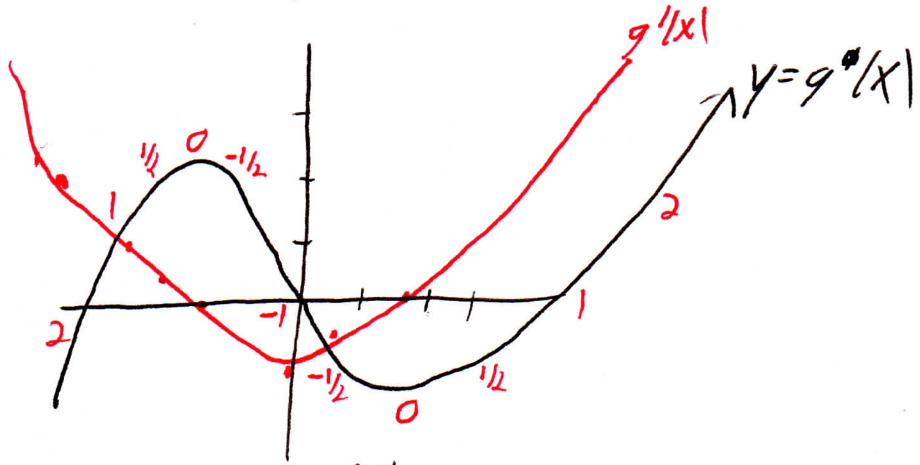
$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3+\cancel{2x}+2h\cancel{-1}-\cancel{x^3}-\cancel{2x}\cancel{+1}}{h}$$

$$= \lim_{h\to 0} 3x^2+2+h(3x+h^2) = 3x^2+2$$

$g'(x)=3x^2+2$ $g'(2)=3\cdot 4+2=14$

$g(2)=13$ $\boxed{y-13=14(x-2)}$

Ex Given graph of $g(x)$ sketch $g'(x)$:



g'

Ex $\lim_{h \to 0} \frac{e^{-2+h} - e^{-2}}{h}$ is $f'(a)$ for some a. Find f & a.

$f(x) = e^x, \quad a = -2$

Ex $g(0) = g(2) = g(4) = 0 \qquad g'(1) = g'(3) = 0$

$g'(0) = g'(4) = 1 \qquad g'(2) = -1 \qquad \lim_{x \to \infty} g(x) = \infty, \ \lim_{x \to -\infty} g(x) = -\infty$

Sketch a possible $g(x)$.

Ex $f(t)$ = # bacteria after t hours in petri dish

a. What is meaning of $f'(5)$? Units?

b. Suppose space & nutrients are unlimited. Which is larger, $f'(5)$ or $f'(10)$?

<u>Notations</u> $y = f(x)$, many notations for derivative:

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}(f(x)) = D_x f(x)!$$

Also if $x(t)$ often write $\dot{x}$ for $\frac{dx}{dt}$.

<u>Recall</u> $f(x) = |x|$ is continuous on $(-\infty, \infty)$ but not differentiable at $x = 0$.

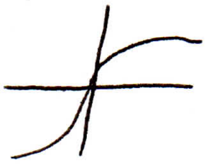
<u>Thm</u> If $f(x)$ is differentiable at $x = a$ then it is continuous:

<u>Proof</u> $$\lim_{x \to a}(f(x) - f(a)) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}(x - a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \to a} x - a$$
$$= f'(a) \cdot 0 = 0 \quad //$$

Rmk Converse false, $y = |x|$

<u>Failure of differentiability</u>

- If $f(x)$ not continuous @ $x = a$ then not diff'ble.
- Corners or kinks
- Tangent line vertical Ex $y = x^{1/3}$



Maple

$$y'(0) = \lim_{h \to 0} \frac{\sqrt[3]{h}}{h} = \lim_{h \to 0} \frac{1}{h^{2/3}} \text{ DNE}$$