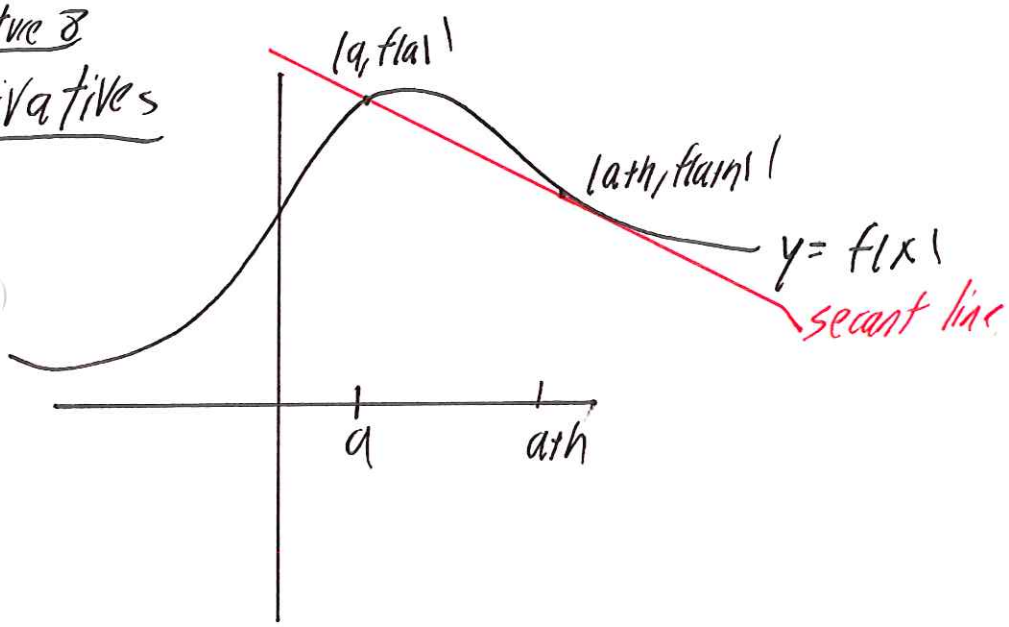
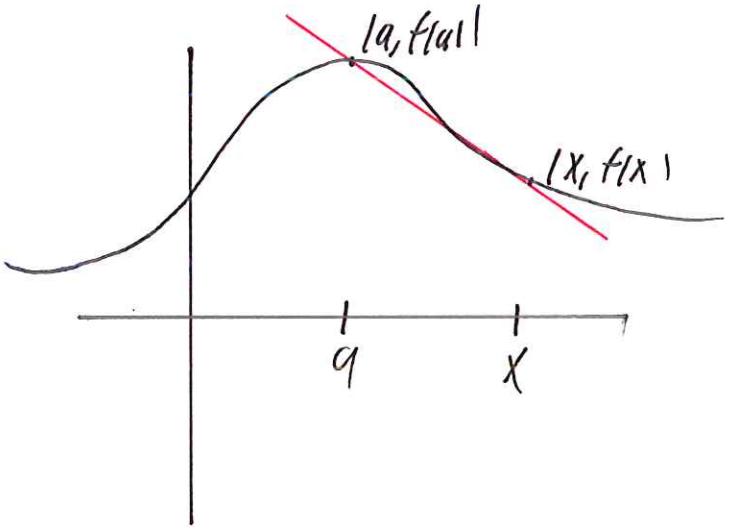
Lecture 8
Derivatives



Key point $\frac{f(a+h)-f(a)}{h}$ is slope of secant line.

a.k.a. avg rate of change of $f(x)$ with respect to x from $x=a$ to $x=a+h$



slope is $\frac{f(x)-f(a)}{x-a}$

Def The derivative of $f(x)$ at $x=a$ is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

if it exists

↳ say $f(x)$ is differentiable at $x=a$

Interpretation

$f'(a)$ is the slope of the tangent line at $(a, f(a))$

$f'(a)$ is the instantaneous rate of change of $f(x)$ with respect to x at $x = a$.

Rate of change of	with respect to	is
position	time	velocity
velocity	time	acceleration
work	time	power
utility	time	marginal utility

Ex $f(x) = x^2$ find tang line at $(3, 9)$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{(3+h)^2 - 9}{h} = \lim_{h\to 0} \frac{h^2 + 6h}{h}$$

$$= \lim_{h\to 0} h + 6 = 6$$

slope $= 6$ point $= (3, 9)$ $\boxed{y - 9 = 6(x - 3)}$

* review point slope

Ex $f(x) = 1/\sqrt{x}$ find $f'(a)$

$$f'(a) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h} = \lim_{h\to 0} \frac{\sqrt{a} - \sqrt{a+h}}{h\sqrt{a}\sqrt{a+h}} \cdot \frac{(\sqrt{a} + \sqrt{a+h})}{\sqrt{a} + \sqrt{a+h}}$$

$$= \lim_{h\to 0} \frac{a - (a+h)}{h\sqrt{a}\sqrt{a+h}(\sqrt{a} + \sqrt{a+h})} = \lim_{h\to 0} \frac{-1}{\sqrt{a}\sqrt{a+h}(\sqrt{a} + \sqrt{a+h})}$$

$$= \frac{-1}{2a^{3/2}} = -\frac{1}{2} a^{-3/2}$$

Derivative as a function

Why calculate $f'(a)$ separately for each a?

Def Let $f(x)$ be a function. The derivative is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ if it exists

Idea Compute some derivatives from scratch, come up with rules for rest.

1. $f(x) = C$ $\quad f'(x) = \lim_{h \to 0} \frac{C - C}{h} = \lim_{h \to 0} \frac{0}{h} = 0$.

2. $f(x) = mx + b$ $\quad f'(x) = m$
   - linear function
   - constant rate of growth

3. $f(x) = x^2$ $\quad f'(x) = 2x$

   Find tang line at $(5, 25)$

4. $f(x) = 1/\sqrt{x}$ $\quad f'(x) = \frac{-1}{2} x^{-3/2}$

5. $f(x) = \sqrt{x}$ $\quad f'(x) = 1/2\sqrt{x}$

6. Is $f(x) = |x|$ differentiable?

7. Estimate $f'(x)$ from graph
   from table