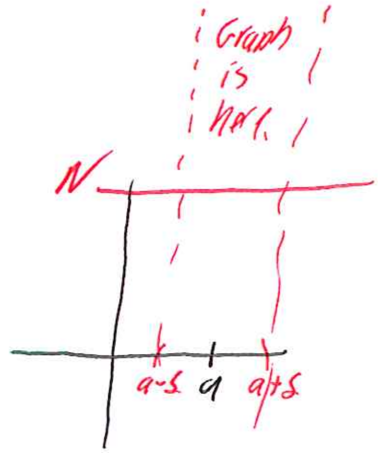
Lecture 4

Review 1-sided limits Ex $\lim_{x\to 0^-} \frac{|x|}{x}$

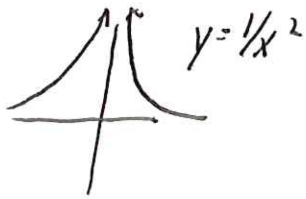
I. Infinite Limits



Def Say $\lim_{x\to a} f(x) = \infty$ if for any $N>0$ there is a δ

so if $0<|x-a|<\delta$ then $f(x)>N$.

Ex $\lim_{x\to 0} \frac{1}{x^2} = \infty$



Exercise Define $\lim_{x\to a} f(x) = -\infty$ and one-sided versions Ex $\lim_{x\to 0^+} \frac{1}{x} = \infty$

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

Definition The graph of $y=f(x)$ has a vertical asymptote at $x=a$

if $\lim_{x\to a^\pm} f(x) = \pm\infty$, so 4 possibilities

Remark Vertical asymptotes often arise when "dividing by 0".

Ex Find $\lim_{x\to 3^-} \frac{2-x^2}{x^2+5x-24}$

Ex Find $\lim_{x\to 2\pi^+} \csc x$.

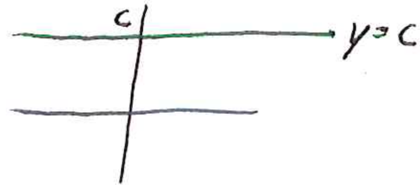
Ex Find $\lim_{x\to 3^-} \frac{x-3}{x^2-2x+3}$

II. Evaluating Limits

* Proving what limits are using the ε-δ definition is hard +

Ex $\lim_{x \to a} x = a.$ Proof Given $\varepsilon > 0$ choose $\delta = \varepsilon$

Ex $\lim_{x \to a} c = c$ Proof Given $\varepsilon > 0$ choose any δ!



Thm (Limit Laws) Suppose $\lim_{x \to a} f(x)$ & $\lim_{x \to a} g(x)$ exist.

1. $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

2. $\lim_{x \to a} c \cdot f(x) = c \cdot \lim_{x \to a} f(x)$

3. $\lim_{x \to a} (f(x) \cdot g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

4. $\lim_{x \to a} f(x)/g(x) = \lim_{x \to a} f(x) \Big/ \lim_{x \to a} g(x)$ if $\lim_{x \to a} g(x) \neq 0.$

* Show proof of #3 on visualizer

* One sided versions left for you.

5. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$

Moral! Often limits are calulated by plugging in:

$$\lim_{x\to 2} 3x^2+5 = \lim_{x\to 2} 3x^2 + \lim_{x\to 2} 5 \quad \text{(law 1)}$$

$$= 3\cdot \lim_{x\to 2} x^2 + 5$$

$$= 3\cdot \lim_{x\to 2} x \lim_{x\to 2} x + 5 = 3\cdot 2\cdot 2+5 = 17$$

Fact
If $f(x)$ is a polynomial or a rational function $p(x)/q(x)$

then $\lim_{x\to a} f(x) = f(a)$ if $q(a)\neq 0$.

Ex $$\lim_{x\to -1} \frac{x^2+2x+3}{x+2} = \frac{2}{1} = 2$$

Ex $$\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$$ "$=\frac{0}{0}$" can conclude nothing.

Rationalize! Multiply by $\frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}$

$$= \lim_{t\to 0} \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)} = \lim_{t\to 0} \frac{1}{\sqrt{t^2+9}+3} = \frac{1}{6}$$

Ex $$f(x) = \begin{cases} \sqrt{x+3} & x \geq -3 \\ x^2-x+C & x \leq -3 \end{cases}$$

Do 2.3 # 2, 10 on visualizer

Find C so that $\lim_{x\to -3} f(x)$ exists