Lecture 37

Review: <u>u substitution</u>

Given $\int f(x)\,dx$
1. Choose $u = g(x)$, $du = g'(x)\,dx$
2. Replace all x's w/ u's to get easier antide.
3. Do integral & put x's back

<u>EX</u>

1. $\int e^{3x}\,dx \qquad u = 3x \quad du = 3dx$

$\qquad\qquad\qquad\qquad \frac{1}{3}du = dx$

$= \int \frac{1}{3} e^{u}\,du = \frac{1}{3}e^{u} + C = \frac{1}{3}e^{3x} + C$

2. $\int \sqrt{1+x^2}\cdot x^5\,dx \qquad u = 1+x^2 \quad \rightarrow \quad x^2 = u-1$

$\qquad\qquad\qquad\qquad du = 2x\,dx$

$= \int \sqrt{1+x^2}\; x^4\, x\,dx = \int \sqrt{u}\,(u-1)^2\cdot \frac{1}{2}\,du$

$= \frac{1}{2}\int \sqrt{u}\,(u^2-2u+1)\,du =$

$= \frac{1}{2}\int u^{5/2} - 2u^{3/2} + u^{1/2}\,du = \frac{1}{2}\left(\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right) + C$

$= \frac{1}{2}\left(\frac{2}{7}(1+x^2)^{7/2} - \frac{4}{5}(1+x^2)^{5/2} + \frac{2}{3}(1+x^2)^{3/2}\right) + C$ //

3. $\int \frac{1}{2-5t}\,dt \qquad u = 2-5t$

$\qquad\qquad\qquad\qquad du = -5dt$

$= -\frac{1}{5}\int \frac{1}{u}\,du = -\frac{1}{5}\ln|u| + C$

$= -\frac{1}{5}\ln|2-5t| + C$

4. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx \qquad u=\sqrt{x} \quad du=\frac{1}{2\sqrt{x}}\,dx$

$= \int 2\sin u\,du = -\cos u + C = -\cos(\sqrt{x}) + C.$

5. $\int \cos(3x)\,dx \qquad u=3x \quad du=3\,dx$

$= \int \frac{1}{3}\cos u\,du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin(3x)$

6. $\int_{0}^{1} \frac{e^2+1}{e^2+z}\,dz \qquad u=e^2+z \quad du=(e^2+1)\,dz$

$= \int \frac{1}{u}\,du = \ln|u|+C = \ln|e^2+z|+C$

Definite Integrals w/ u-substitution

Ex $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$ really means $\int_{x=1}^{x=2} \frac{e^{1/x}}{x^2}\,dx$

$u=1/x \quad du=-1/x^2\,dx \qquad x=1 \rightarrow u=1$

$x=2 \rightarrow u=1/2$

$\int_{u=1}^{u=1/2} -e^u\,du = -e^u\Big|_{u=1}^{u=1/2} = -(e^{1/2}-e^1) = e^1-e^{1/2}$

$= e-\sqrt{e}$

OR Complete indef int, put x's back

$\int \frac{e^{1/x}}{x^2}\,dx = -e^{1/x}$ then sub in

$-e^{1/x}\Big|_1^2 = -e^{1/2}+e^1 = e-\sqrt{e}$

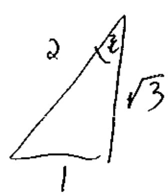
<u>Written out</u>

$u=g(x)$ $\displaystyle\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$

<u>Ex</u> $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ $u=\ln x$ $du=\frac{1}{x}\,dx$ $x=e \to u=1$
$x=e^4 \to u=4$

$$= \int_1^4 \frac{1}{\sqrt{u}}\,du = 2u^{1/2}\Big|_1^4 = 2\sqrt{4}-2\sqrt{1}$$
$$= 2$$

<u>Ex</u> $\displaystyle\int_0^{\pi/6} \frac{\sin t}{\cos^2 t}\,dt$ $u=\cos t$ $du=-\sin t\,dt$



$t=0 \to u=1$
$t=\pi/6$ $u=\sqrt{3}/2$

$$= \int_1^{\sqrt{3}/2} -\frac{1}{u^2}\,du = \int_1^{3/2} -u^{-2}\,du = u^{-1}\Big|_1^{3/2} = \frac{2}{3}-1 = -1/3$$

<u>Symmetry Thm</u> Suppose f is continuous on $[-a,a]$

1. If f is even then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$
2. If f is odd then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$

<u>Pf</u> $\displaystyle\int_{-a}^{a} f(x)\,dx = \int_0^a f(x)\,dx + \int_{-a}^{0} f(x)\,dx.$ Suppose $f(x)=f(-x)$

$u=-x$ $du=-dx$

$\displaystyle\int_0^a f(-u)\,du = \int_0^a f(u)\,du$ //