Lecture 36

Review Indefinite integral! $\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$

Ex $\int x^3 dx = \frac{1}{4}x^4 + C$

Rmk 1 Every differentiation formula has corresponding antider formula.

2 $\int f(x)\,dx$ is a function, $\int_a^b f(x)\,dx$ is a number.

FTOC If $F'(x) = f(x)$ then $\int_a^b f(x)\,dx = F(b) - F(a)$. This is useless unless we can find $F(x)$!

Ex

$$\int x^3 - \sec^2 x\,dx = \frac{1}{4}x^4 - \tan x + C$$

$$\int_0^3 \frac{1}{1+x^2}\,dx = \tan^{-1}x\Big|_0^3 = \tan^{-1}3 - \tan^{-1}0 = \boxed{\tan^{-1}3}$$

Consider: $\int_a^b F'(x)\,dx = F(b) - F(a)$

Conclude The integral of a rate of change = the net change.

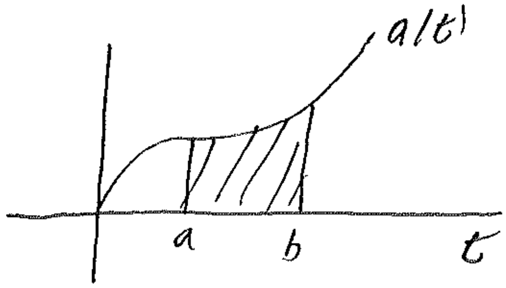
Ex $C(x)$ = cost to produce x units, $C'(x)$ = marginal cost

$\int_{x_1}^{x_2} C'(x)\,dx$ = increase in cost as production goes from x_1 to x_2

Ex $s(t)$ = position at time t, $v(t) = s'(t)$

$\int_a^b v(t)\,dt$ = net change in posititiun.

Ex



area = net change in $v(t)$ as t goes a to b

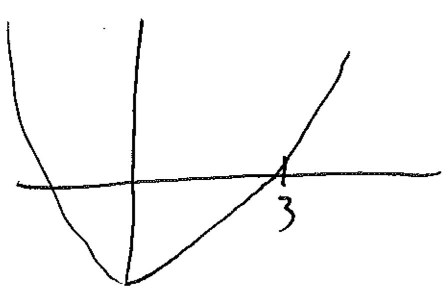
Ex particle moves along a line, $v(t) = t^2 - t - 6$

1. Find displacement $1 \le t \le 4$

2. Find distance travelled

A: 1. $\int_1^4 v(t)\,dt = \frac{t^3}{3} - \frac{t^2}{2} - 6t \Big|_1^4 = \left(\frac{64}{3} - 8 - 24\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$

2.



$v(t) = (t-3)(t+2)$

distance $= \int_1^4 |\vec{v}(t)|\,dt$

$= \int_1^3 -v(t)\,dt + \int_3^4 v(t)\,dt$

Ex $f(x)$ = slope of a trail at distance x miles from start. What does $\int_1^3 f(x)dx$ represent?

A $f(x)$ = rate of change of elevation so $\int_1^3 f(x)dx$ is net change in elevation from $x=1$ to $x=3$

5.5 u-substitution

Consider $\int 2xe^{x^2}dx = e^{x^2}+C$, can do in head, want to formalize this

Recall $\frac{d}{dx}e^u = e^u \frac{du}{dx}$

$\int 2xe^{x^2}dx$ Let $u=x^2$
$du = 2x\,dx$

$\parallel$

$\int e^u du = e^u + C = e^{x^2}+C$

Idea Substitute $u=g(x)$
$du = g'(x)dx$.

Goal Turn antiderivative into one on our list.

Ex $\int \sin^4 x \cos x \, dx$ $\quad u = \sin x$, $du = \cos x \, dx$

$$= \int u^4 \, du = \frac{u^5}{5} + C = \frac{\sin^5 x}{5} + C$$

Rmk Easy to check!

Ex $\int \sqrt{3x-5} \, dx \quad u = 3x-5 \quad du = 3\,dx \rightarrow \frac{1}{3} du = dx$

$$= \int \sqrt{u} \cdot \frac{1}{3} du = \frac{1}{3} \cdot \frac{u^{3/2}}{3/2} = \frac{2}{9} u^{3/2} + C = \frac{2}{9}(3x-5)^{3/2} + C$$

Ex Recall $\int \frac{1}{u} du = \ln|u| + C$

Consider $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

Let $u = \cos x \quad du = -\sin x \, dx$

$$= \int -\frac{1}{u} du = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$= \ln|\sec x| + C$$

$$\boxed{\int \tan x = \ln|\sec x| + C}$$